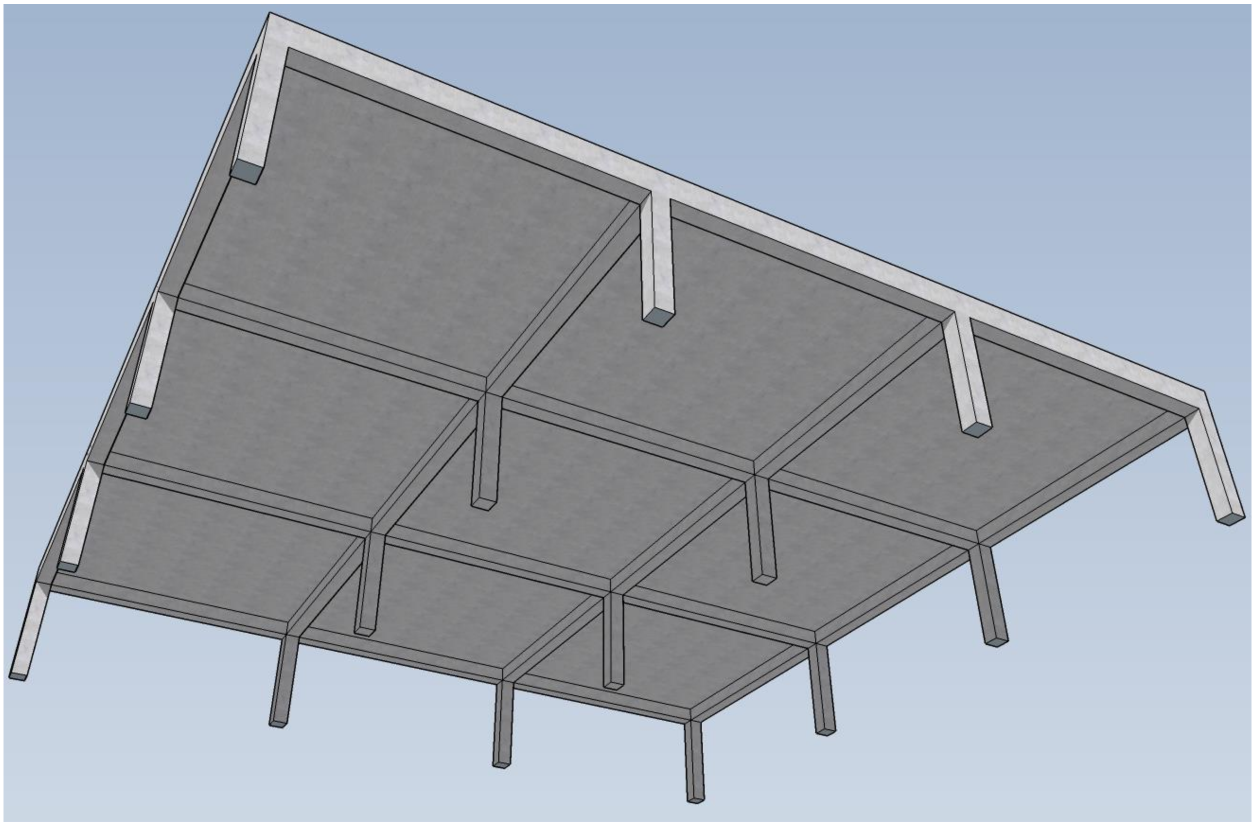
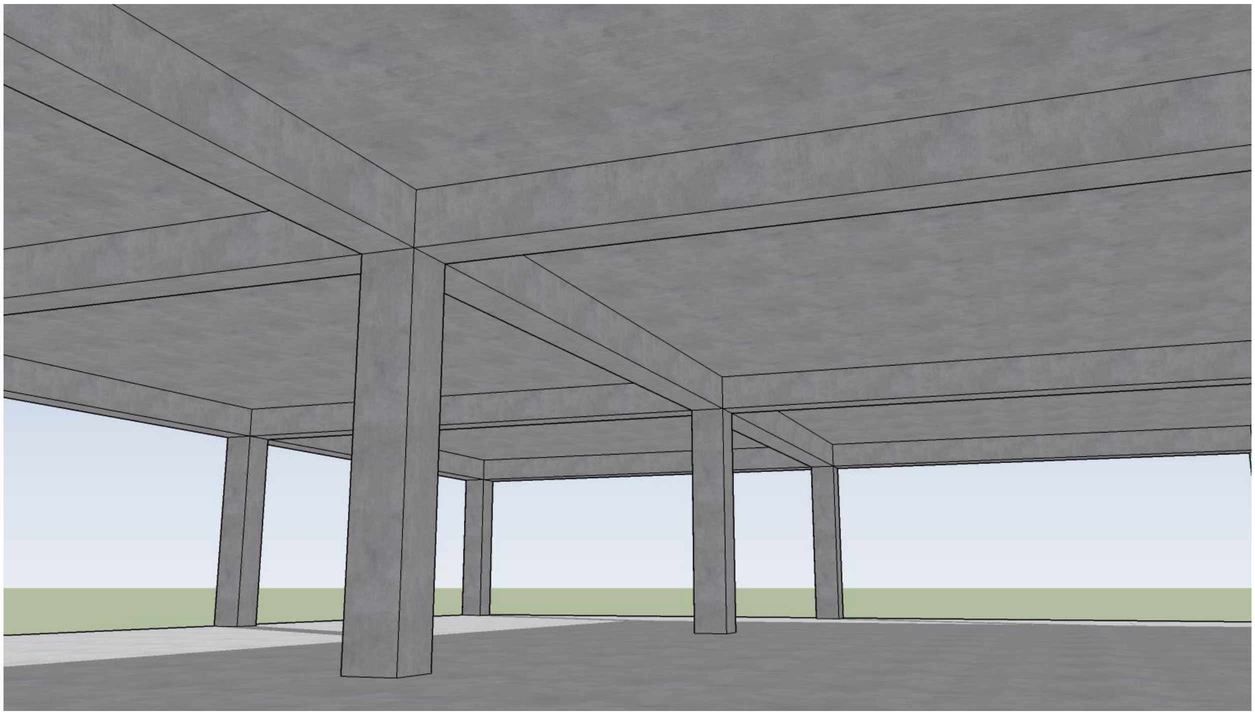
Two-Way Concrete Floor Slab with Beams Design and Detailing (CSA A23.3-14)





Two-Way Concrete Floor Slab with Beams Design and Detailing (CSA A23.3-14)

Design the slab system shown in Figure 1 for an intermediate floor where the story height = 3.7 m, column cross-sectional dimensions = 450 mm × 450 mm, edge beam dimensions = 350 mm × 700 mm, interior beam dimensions = 350 mm x 500 mm, and unfactored live load = 4.8 kN/m^2. The lateral loads are resisted by shear walls. Normal weight concrete with ultimate strength (f_c'= 25 MPa) is used for all members, respectively. And reinforcement with F_y = 400 MPa is used. Use the Elastic Frame Method (EFM) and compare the results with spSlab model results.

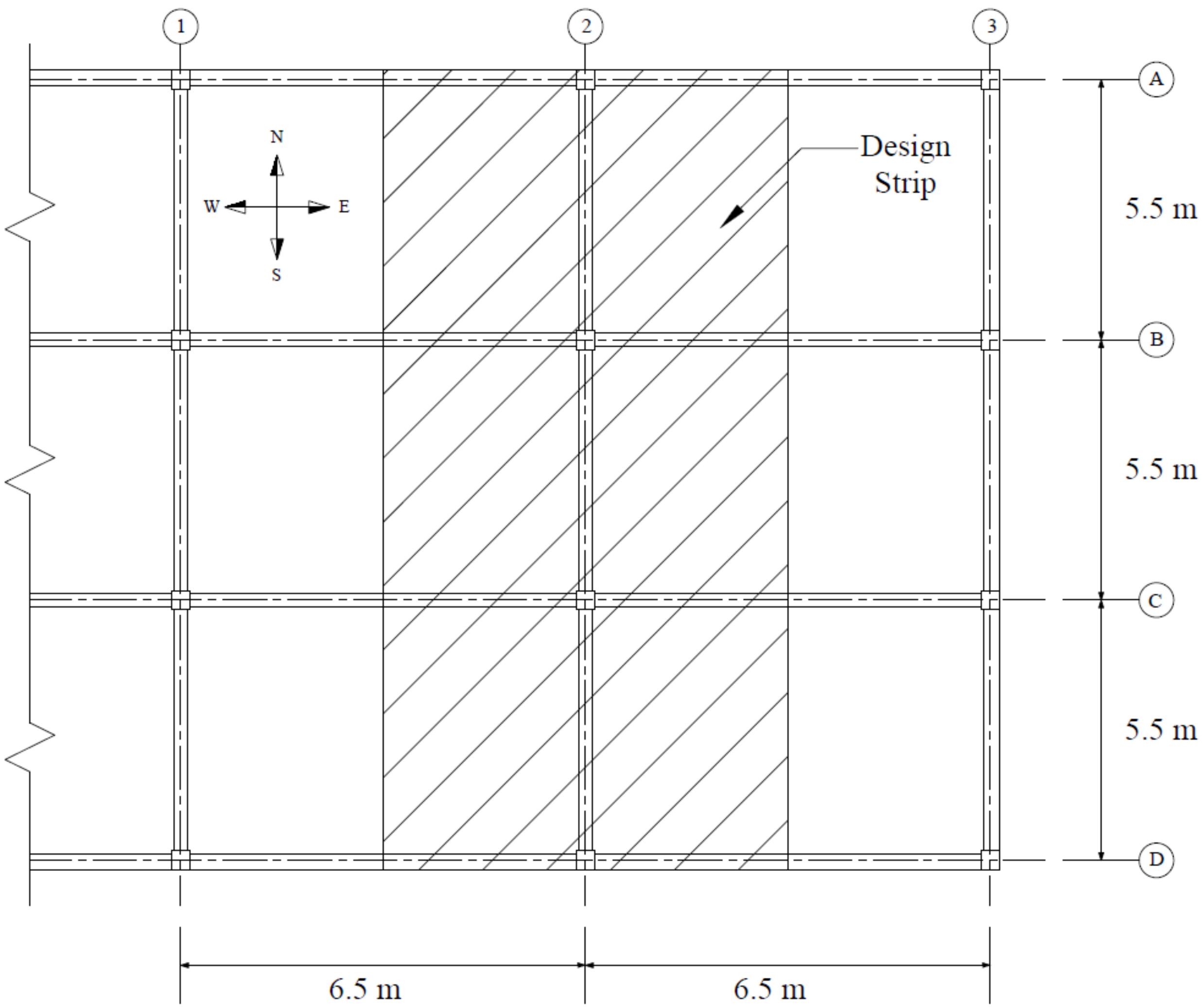


Figure 1 – Two-Way Slab with Beams Spanning between all Supports

Contents

Code

Design of Concrete Structures (CSA A23.3-14) and Explanatory Notes on CSA Group standard A23.3-14 "Design of Concrete Structures"

References

CAC Concrete Design Handbook, 4th Edition, Cement Association of Canada

Notes on ACI 318-11 Building Code Requirements for Structural Concrete, Twelfth Edition, 2013 Portland Cement Association.

Design Data

Floor-to-Floor Height = 3.7 m (provided by architectural drawings)

Columns = 450 × 450 mm

Interior beams = 350 × 500 mm

Edge beams = 350 × 700 mm

w_c = 24 kN/m^3

f_c' = 25 MPa

f_y = 400 MPa

Live load, L_o = 4.8 kN/m^2

Solution

1. Preliminary Slab Thickness Sizing

Control of deflections. ***CSA A23.3 (13.2.5)***

In lieu of detailed calculation for deflections, CSA A23.3 Code gives minimum thickness for two-way slab with beams between all supports on all sides in ***Clause 13.2.5***.

Ratio of moment of inertia of beam section to moment of inertia of a slab (α) is computed as follows:

$$\alpha = \frac{I_b}{I_s}$$ ***CSA A23.3 (13.2.5)***

The moment of inertia for the effective beam and slab sections can be calculated as follows:

$$I_b = \frac{b_w h^3}{12}\left(2.5\left(1-\frac{h_s}{h}\right)\right)$$ ***CSA A23.3 (Eq. 13.4)***

The preliminary thickness of 155 mm is assumed and it will be checked in next steps.

Edge Beams:

The effective beam and slab sections for the computation of stiffness ratio for edge beam is calculated as follows:

For North-South Edge Beams:

$$I_b = \frac{350\times700^3}{12}\left(2.5\times\left(1-\frac{155}{700}\right)\right)=1.95\times10^{10}\ \text{mm}^4$$

$$I_s = \frac{6,500\times155^3}{12} = 2.02\times10^9\ \text{mm}^4$$

$$\alpha = \frac{1.95\times10^{10}}{2.02\times10^9} = 9.65$$

For East-West Edge Beams:

$$I_b = \frac{350\times700^3}{12}\left(2.5\times\left(1-\frac{155}{700}\right)\right)=1.95\times10^{10}\ \text{mm}^4$$

$$.\ I_s = \frac{5,500\times155^3}{12} = 1.71\times10^9\ \text{mm}^4\ .$$

$$\alpha = \frac{1.95\times10^{10}}{1.71\times10^9} = 11.41$$

Interior Beams:

For North-South Interior Beams:

$$I_b = \frac{350\times500^3}{12}\left(2.5\times\left(1-\frac{155}{500}\right)\right)=6.29\times10^9\ \text{mm}^4$$

$$\alpha = \frac{6.29\times10^9}{2.02\times10^9} = 3.12$$

For East-West Interior Beams:

$$I_b = \frac{350\times500^3}{12}\left(2.5\times\left(1-\frac{155}{500}\right)\right)=6.29\times10^9\ \text{mm}^4$$

$$\alpha = \frac{6.29\times10^9}{1.71\times10^9} = 3.68$$

The average of α for the beams on four sides of exterior and interior panels are calculated as:

For exterior panels: $\alpha_m = \dfrac{(11.41+3.68+3.12+3.12)}{4} = 5.33$

For interior panels: $\alpha_m = \dfrac{(2\times3.68+2\times3.12)}{4} = 3.40$

α_m shall not be taken greater than 2.0, then α_m = 2.0 for both exterior and interior panels.

The minimum slab thickness is given by:

$$h_{\min} = \frac{l_n\left(0.6 + \dfrac{f_y}{1,000}\right)}{30 + 4\beta\alpha_m}$$ ***CSA A23.3-14 (13.2.5)***

Where:

l_n = clear span in the long direction measured face to face of columns = 6.05 m = 6050 mm

$$\beta = \frac{\text{clear span in the long direction}}{\text{clear span in the short direction}} = \frac{6500 - 450}{5500 - 450} = 1.182$$

$$h_{\min} = \frac{6,050\left(0.6 + \dfrac{400}{1,000}\right)}{30 + 4 \times 1.182 \times 2}$$

The assumed thickness is more than the h_{min}. Use 155 mm slab thickness.

2. Two-Way Slab Analysis and Design – Using Elastic Frame Method (EFM)

EFM (as known as Equivalent Frame Method in the ACI 318) is the most comprehensive and detailed procedure provided by the CSA A23.3 for the analysis and design of two-way slab systems where these systems may, for purposes of analysis, be considered a series of plane frames acting longitudinally and transversely through the building. Each frame shall be composed of equivalent line members intersecting at member centerlines, shall follow a column line, and shall include the portion of slab bounded laterally by the centerline of the panel on each side. ***CSA A23.3-14 (13.8.1.1)***

Probably the most frequently used method to determine design moments in regular two-way slab systems is to consider the slab as a series of two-dimensional frames that are analyzed elastically. When using this analogy, it is essential that stiffness properties of the elements of the frame be selected to properly represent the behavior of the three-dimensional slab system.

In a typical frame analysis it is assumed that at a beam-column connection all members meeting at the joint undergo the same rotation. For uniform gravity loading this reduced restraint is accounted for by reducing the effective stiffness of the column by either Clause 13.8.2 or Clause 13.8.3. ***CSA A23.3-14 (N.13.8)***

Each floor and roof slab with attached columns may be analyzed separately, with the far ends of the columns considered fixed. ***CSA A23.3-14 (13.8.1.2)***

The moment of inertia of column and slab-beam elements at any cross-section outside of joints or column capitals shall be based on the gross area of concrete at that section. ***CSA A23.3-14 (13.8.2.5)***

An equivalent column shall be assumed to consist of the actual columns above and below the slab- beam plus an attached torsional member transverse to the direction of the span for which moments are being determined. ***CSA A23.3-14 (13.8.2.5)***

2.1. Elastic frame method limitations

In EFM, live load shall be arranged in accordance with 13.8.4 which requires slab systems to be analyzed and designed for the most demanding set of forces established by investigating the effects of live load placed in various critical patterns. ***CSA A23.3-14 (13.8.4)***

Complete analysis must include representative interior and exterior elastic frames in both the longitudinal and transverse directions of the floor. ***CSA A23.3-14 (13.8.1.1)***

Panels shall be rectangular, with a ratio of longer to shorter panel dimensions, measured center-to-center of supports, not to exceed 2. ***CSA A23.3-14 (3.1a)***

For slab systems with beams between supports, the relative effective stiffness of beams in the two directions is not less than 0.2 or greater than 2. ***CSA A23.3-14 (3.1b)***

Column offsets are not greater than 20% of the span (in the direction of offset) from either axis between centerlines of successive columns. ***CSA A23.3-14 (3.1c)***

The reinforcement is placed in an orthogonal grid. ***CSA A23.3-14 (3.1d)***

2.2. Frame members of elastic frame

Determine moment distribution factors and fixed-end moments for the elastic frame members. The moment distribution procedure will be used to analyze the elastic frame. Stiffness factors k , carry over factors COF, and fixed-end moment factors FEM for the slab-beams and column members are determined using the design aids tables at ***Appendix 20A of PCA Notes on ACI 318-11***. These calculations are shown below.

a. Flexural stiffness of slab-beams at both ends, K_{sb}.

$$\frac{c_{N1}}{\ell_1} = \frac{450}{5{,}500} = 0.082, \quad \frac{c_{N2}}{\ell_2} = \frac{450}{6{,}500} = 0.069$$

For $c_{F1} = c_{F2}$ stiffness factors, $k_{NF} = k_{FN} = 4.15$ — ***PCA Notes on ACI 318-11 (Table A1)***

Thus, $K_{sb} = k_{NF}\dfrac{E_c I_{sb}}{\ell_1} = 4.15\dfrac{E_c I_{sb}}{\ell_1}$ — ***PCA Notes on ACI 318-11 (Table A1)***

Where I_{sb} is the moment of inertia of slab-beam section shown in Figure 2 and can be computed with the aid of Figure 3 as follows:

$$I_{sb} = C_t\left(\frac{b_w h^3}{12}\right) = 2.72\left(\frac{350 \times 500^3}{12}\right) = 9.92 \times 10^9 \text{ mm}^4$$

$$K_{sb} = 4.15\frac{E_c \times 9.92 \times 10^9}{5{,}500} = 7.48 \times 10^3 \ E_c \text{ N.m}$$

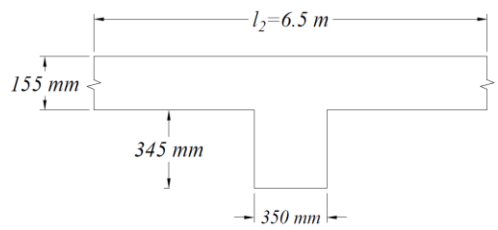


Figure 2 – Cross-Section of Slab-Beam

Carry-over factor COF = 0.508 — ***PCA Notes on ACI 318-11 (Table A1)***

Fixed-end moment FEM $= 0.0844 w_u \ell_2 \ell_1^{\,2}$ — ***PCA Notes on ACI 318-11 (Table A1)***

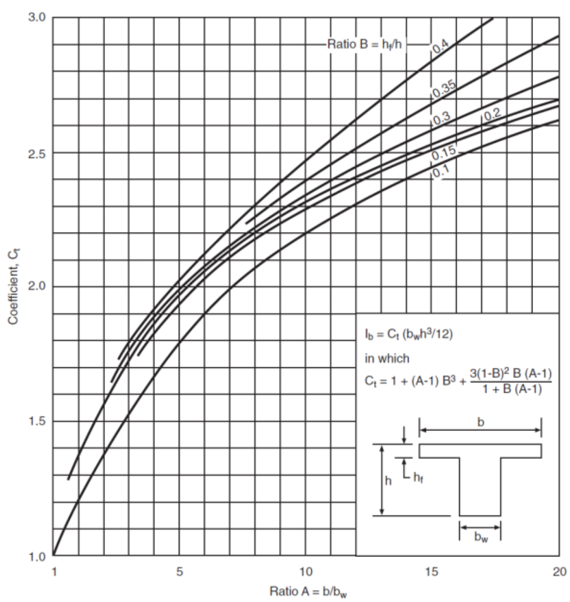


Figure 3 – Coefficient C_t for Gross Moment of Inertia of Flanged Sections

b. Flexural stiffness of column members at both ends, K_c.

Referring to ***Table A7, Appendix 20A***:

For Interior Columns:

$t_a = 500 - 155/2 = 422.5 \text{ mm}$, $t_b = 77.5 \text{ mm}$

$H = 3.7 \text{ m} = 3700 \text{ mm},\ H_c = 3700 - 500 = 3200 \text{ mm},\ \frac{t_a}{t_b} = 5.45,\ \frac{H}{H_c} = 1.16$

Thus, $k_{c,\ top} = 6.55$ and $k_{c,\ bottom} = 4.91$ by interpolation.

$I_c = \frac{c^4}{12} = \frac{(450)^4}{12} = 3.42 \times 10^9 \text{ mm}^4$

$\ell_c = 3.7 \text{ m} = 3,700 \text{ mm}$

$K_c = \frac{k_c E_{cc} I_c}{\ell_c}$ ***PCA Notes on ACI 318-11 (Table A7)***

$K_{c,\ top} = \frac{6.55 \times 3.42 \times 10^9 \times E_c}{3,700} = 6.05 \times 10^3 E_c \text{ N.m}$

$K_{c,\ bottom} = \frac{4.915 \times 3.42 \times 10^9 \times E_c}{3,700} = 4.54 \times 10^3 E_c \text{ N.m}$

For Exterior Columns:

$t_a = 700 - 155/2 = 622.5 \text{ mm}$, $t_b = 77.5 \text{ mm}$

$H = 3.7 \text{ m} = 3,700 \text{ mm},\ H_c = 3,700 - 700 = 3,000 \text{ mm},\ \frac{t_a}{t_b} = 8.0,\ \frac{H}{H_c} = 1.23$

Thus, $k_{c,\ top} = 8.45$ and $k_{c,\ bottom} = 5.47$ by interpolation.

$I_c = \frac{c^4}{12} = \frac{(450)^4}{12} = 3.42 \times 10^9 \text{ mm}^4$

$\ell_c = 3.7 \text{ ft} = 3,700 \text{ mm}$

$K_c = \frac{k_c E_{cc} I_c}{\ell_c}$ ***PCA Notes on ACI 318-11 (Table A7)***

$K_{c,\ top} = \frac{8.45 \times 3.42 \times 10^9 \times E_c}{3,700} = 7.80 \times 10^3 E_c \text{ N.m}$

$K_{c,\ bottom} = \frac{5.47 \times 3.42 \times 10^9 \times E_c}{3,700} = 5.05 \times 10^3 E_c$

c. Torsional stiffness of torsional members, K_t.

$$K_t = \sum \frac{9E_{cs}C}{[\ell_t(1-\frac{c_2}{\ell_t})^3]}$$ **CSA A23.3-14 (13.8.2.8)**

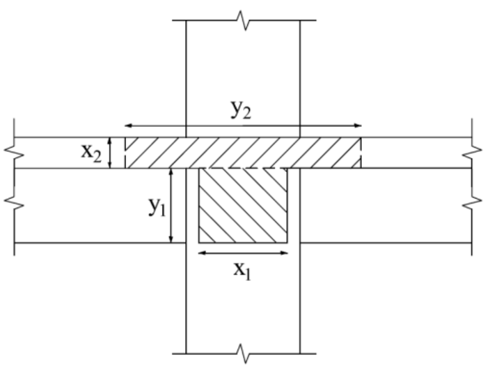
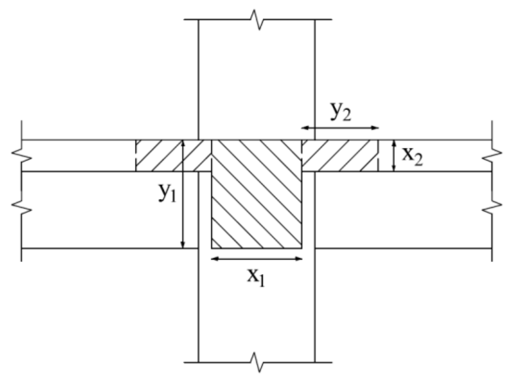
For Interior Columns:

$$K_t = \frac{9E_c \times 4.61\times10^9}{5,500(0.918)^3} = 9.74\times10^3 E_c \text{ N.m}$$

Where:

$$1-\frac{c_2}{\ell_t} = 1-\frac{450}{5,500} = 0.918$$

$$C = \sum\left(1-0.63\frac{x}{y}\right)\left(\frac{x^3y}{3}\right)$$ **CSA A23.3-14 (13.8.2.9)**

x_1 = 350 mm, y_1 = 345 mm, $C_1 = 1.78\times10^9$; x_2 = 155 mm, y_2 = 1,040 mm, $C_2 = 1.17\times10^9$	x_1 = 350 mm, y_1 = 500 mm, $C_1 = 3.99\times10^9$; x_2 = 150 mm, y_2 = 345 mm, $C_2 = 3.08\times10^8$
$\sum C = 1.78\times10^9 + 1.17\times10^9 = 2.95\times10^9$ mm^4	$\sum C = 3.99\times10^9 + 3.07\times10^8 = 4.61\times10^9$ mm^4
	

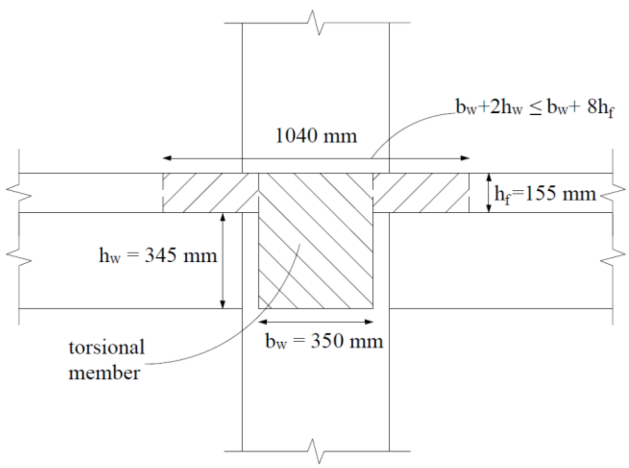


Figure 4 – Attached Torsional Member at Interior Column

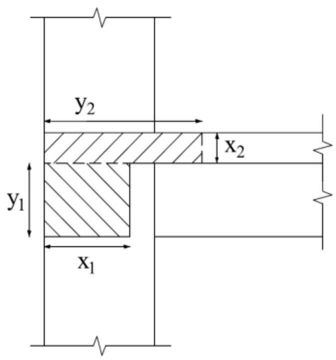
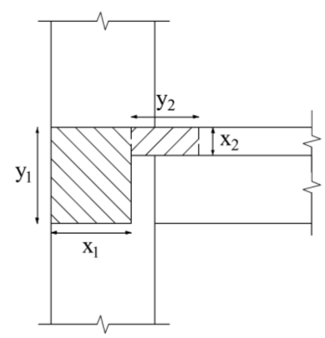
For Exterior Columns:

$$K_t = \frac{9E_c \times 7.41\times10^9}{5,500(0.918)^3} = 1.57\times10^4 E_c \text{ N.m}$$

Where:

$$1-\frac{c_2}{\ell_t} = 1-\frac{450}{5,500} = 0.918$$

$$C = \sum\left(1-0.63\frac{x}{y}\right)\left(\frac{x^3y}{3}\right)$$ ***CSA A23.3-14 (13.8.2.9)***

x_1 = 350 mm y_1 = 545 mm $C_1 = 4.64\times10^9$	x_2 = 155 mm y_2 = 895 mm $C_2 = 9.90\times10^8$	x_1 = 350 mm y_1 =700 mm $C_1 = 6.85\times10^9$	x_2 = 155 mm y_2 = 545 mm $C_2 = 5.55\times10^9$
$\sum C = 4.64\times10^9 + 9.90\times10^8 = 5.63\times10^9$ mm^4		$\sum C = 6.85\times10^9 + 5.55\times10^9 = 7.41\times10^9$ mm^4	
			

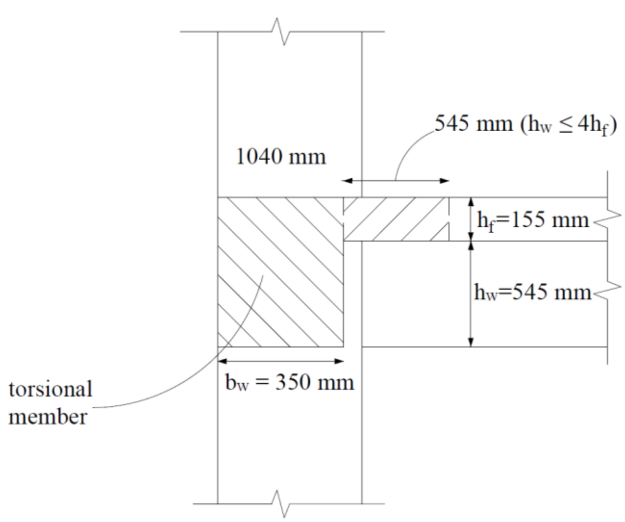


Figure 5 – Attached Torsional Member at Exterior Column

d. Increased torsional stiffness due to parallel beams, K_{ta}.

For Interior Columns:

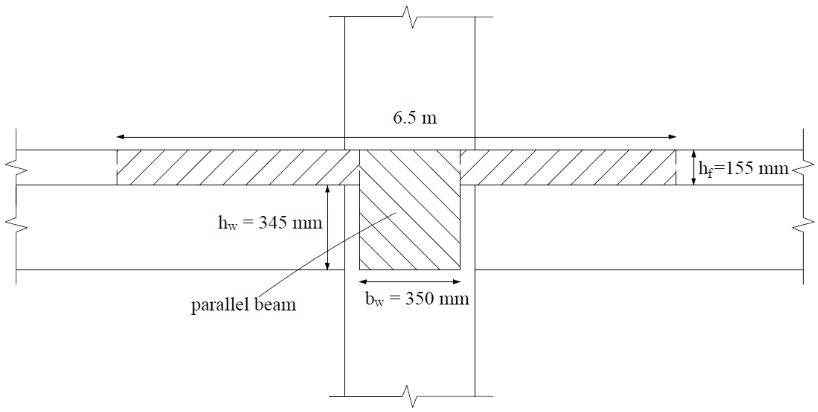


Figure 6 – Slab-Beam in the Direction of Analysis

$$K_{ta} = \frac{K_t I_{sb}}{I_s} = \frac{9.74\times10^3 E_c \times 9.92\times10^9}{2.02\times10^9} = 4.79\times10^4 E_c \text{ N.m}$$

Where:

$$I_s = \frac{l_2 \times h^3}{12} = \frac{6,500\times155^3}{12} = 2.02\times10^9 \text{ mm}^4$$

For Exterior Columns:

$$K_{ta} = \frac{K_t I_{sb}}{I_s} = \frac{1.57\times10^4 E_c \times 9.92\times10^9}{2.02\times10^9} = 7.70\times10^4 E_c \text{ N.m}$$

e. Equivalent column stiffness K_{ec}.

$$K_{ec} = \frac{\sum K_c \times \sum K_{ta}}{\sum K_c + \sum K_{ta}}$$

Where $\sum K_{ta}$ is for two torsional members one on each side of the column, and $\sum K_c$ is for the upper and lower columns at the slab-beam joint of an intermediate floor.

For Interior Columns:

$$K_{ec} = \frac{(6.05\times10^3 E_c + 4.54\times10^3 E_c)(2\times4.79\times10^4 E_c)}{(6.05\times10^3 E_c + 4.54\times10^3 E_c) + (2\times4.79\times10^4 E_c)} = 9.53\times10^3 E_c$$

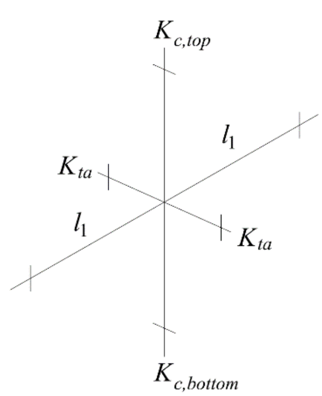


Figure 7 – Equivalent Column Stiffness

For Exterior Columns:

$$K_{ec}=\frac{(7.80\times10^3E_c+5.05\times10^3E_c)(2\times7.70\times10^4E_c)}{(7.80\times10^3E_c+5.05\times10^3E_c)+(2\times7.70\times10^4E_c)}=1.19\times10^4E_c$$

f. Slab-beam joint distribution factors, *DF*.

At exterior joint,

$$DF=\frac{7.48\times10^3E_c}{(7.48\times10^3E_c+1.19\times10^4E_c)}=0.387$$

At interior joint,

$$DF=\frac{7.48\times10^3E_c}{(7.48\times10^3E_c+9.53\times10^3E_c)}=0.305$$

COF for slab-beam =0.508

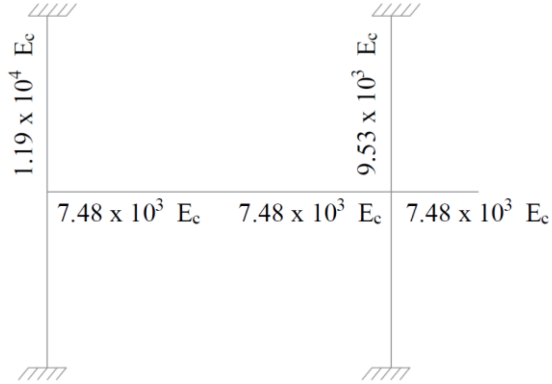


Figure 8 – Slab and Column Stiffness

2.3. Elastic frame analysis

Determine negative and positive moments for the slab-beams using the moment distribution method.

With an unfactored live-to-dead load ratio:

$$\frac{L}{D}=\frac{4.8}{(24\times155/1000)}=1.29>\frac{3}{4}$$

The frame will be analyzed for five loading conditions with pattern loading and partial live load as allowed by ***CSA A23.3-14 (13.8.4).***

a. Factored load and Fixed-End Moments (FEM's).

Factored dead load $w_{df}=1.25(3.72+0.446)=5.21\text{ kN/m}^2$

Where (0.446 kN/m^2 = (0.345 x 0.35) × 24 / 6.5 is the weight of beam stem per foot divided by l_2)

Factored live load $w_{Lf}=1.5(4.8)=7.2\text{ kN/m}^2$

Factored load $w_f=w_{Df}+w_{Lf}=12.41\text{ kN/m}^2$

FEM's for slab-beam $=m_{NF}w_f\ell_2\ell_1^{\,2}$ ***PCA Notes on ACI 318-11 (Table A1)***

FEM due to $w_{Df}+w_{Lf}=0.0844\times(12.41\times6.5)\times5.5^2=206.02\text{ kN.m}$

FEM due to $w_{Df}+\frac{3}{4}w_{Lf}=0.0844\times(10.61\times6.5)\times5.5^2=176.13\text{ kN.m}$

FEM due to $w_{Df}=0.0844\times(5.21\times6.5)\times5.5^2=86.47\text{ kN.m}$

b. Moment distribution.

Moment distribution for the five loading conditions is shown in Table 1 (The unit for moment values is kN.m). Counter-clockwise rotational moments acting on member ends are taken as positive. Positive span moments are determined from the following equation:

$$M_{u(midspan)} = M_o - \frac{(M_{uL} + M_{uR})}{2}$$

Where M_o is the moment at the midspan for a simple beam.

When the end moments are not equal, the maximum moment in the span does not occur at the midspan, but its value is close to that midspan for this example.

Positive moment in span 1-2 for loading (1):

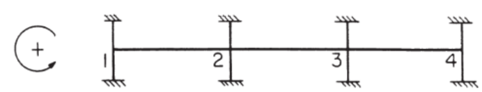
$$M_f^+ = (12.41 \times 6.5)\frac{5.5^2}{8} - \frac{(131.1 + 232.8)}{2} = 123.0 \text{ kN.m}$$

Positive moment span 2-3 for loading (1):

$$M_f^+ = (12.41 \times 6.5)\frac{5.5^2}{8} - \frac{(213.5 + 213.5)}{2} = 91.5 \text{ kN.m}$$

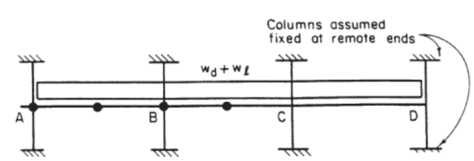
Table 1 – Moment Distribution for Partial Frame (Transverse Direction)

Joint	1	2		3		4
Member	1-2	2-1	2-3	3-2	3-4	4-3
DF	0.387	0.305	0.305	0.305	0.305	0.387
COF	0.508	0.508	0.508	0.508	0.508	0.508



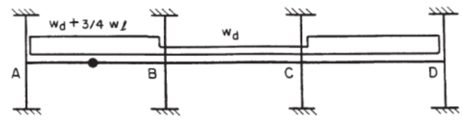
Loading (1) All spans loaded with full factored live load

FEM	206.0	-206.0	206.0	-206.0	206.0	-206.0
Dist	-79.7	0.0	0.0	0.0	0.0	79.7
CO	0.0	-40.5	0.0	0.0	40.5	0.0
Dist	0.0	12.4	12.4	-12.4	-12.4	0.0
CO	6.3	0.0	-6.3	6.3	0.0	-6.3
Dist	-2.4	1.9	1.9	-1.9	-1.9	2.4
CO	1.0	-1.2	-1.0	1.0	1.2	-1.0
Dist	-0.4	0.7	0.7	-0.7	-0.7	0.4
CO	0.3	-0.2	-0.3	0.3	0.2	-0.3
Dist	-0.1	0.2	0.2	-0.2	-0.2	0.1
CO	0.1	-0.1	-0.1	0.1	0.1	-0.1
Dist	0.0	0.1	0.1	-0.1	-0.1	0.0
M	131.1	-232.8	213.5	-213.5	232.8	-131.1
Midspan M	123.0		91.5		123.0	



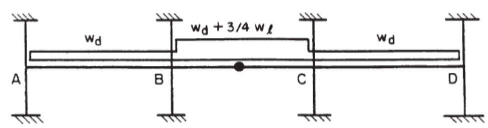
(1) Loading pattern for design moments in all spans with L ≤ 3/4 D

Loading (2) First and third spans loaded with 3/4 factored live load						
FEM	176.1	-176.1	86.5	-86.5	176.1	-176.1
Dist	-68.1	27.4	27.4	-27.4	-27.4	68.1
CO	13.9	-34.6	-13.9	13.9	34.6	-13.9
Dist	-5.4	14.8	14.8	-14.8	-14.8	5.4
CO	-2.9	3.1	3.1	-3.1	-3.1	2.9
Dist	1.6	-1.5	-1.6	1.6	1.5	-1.6
CO	-0.6	0.9	0.9	-0.9	-0.9	0.6
Dist	0.5	-0.3	-0.5	0.5	0.3	-0.5
CO	-0.2	0.2	0.2	-0.2	-0.2	0.2
Dist	0.1	-0.1	-0.1	0.1	0.1	-0.1
CO	-0.1	0.1	0.1	-0.1	-0.1	0.1
Dist	0.0	0.0	0.0	0.0	0.0	0.0
M	122.6	-168.8	109.4	-109.4	168.8	-122.6
Midspan M	115.0		18.6		115.0	



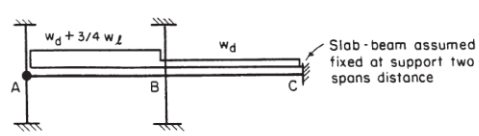
(2) Loading pattern for positive design moment in span AB*

Loading (3) Center span loaded with 3/4 factored live load						
FEM	86.5	-86.5	176.1	-176.1	86.5	-86.5
Dist	-33.4	-27.4	-27.4	27.4	27.4	33.4
CO	-13.9	-17.0	13.9	-13.9	17.0	13.9
Dist	5.4	0.9	0.9	-0.9	-0.9	-5.4
CO	0.5	2.7	-0.5	0.5	-2.7	-0.5
Dist	-0.2	-0.7	-0.7	0.7	0.7	0.2
CO	-0.4	-0.1	0.4	-0.4	0.1	0.4
Dist	0.1	-0.1	-0.1	0.1	0.1	-0.1
CO	0.0	0.1	0.0	0.0	-0.1	0.0
Dist	0.0	0.0	0.0	0.0	0.0	0.0
M	44.6	-128.0	162.7	-162.7	128.0	-44.6
Midspan M	41.7		98.0		41.7	

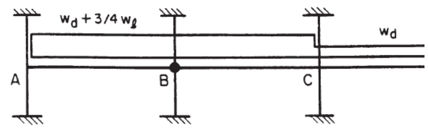


(3) Loading pattern for positive design moment in span BC*

Loading (4) First span loaded with 3/4 factored live load and beam-slab assumed fixed at support two spans away				
FEM	176.1	-176.1	86.5	-86.5
Dist	-68.1	27.4	27.4	0.0
CO	13.9	-34.6	0.0	13.9
Dist	-5.4	10.6	10.6	0.0
CO	5.4	-2.7	0.0	5.4
Dist	-2.1	0.8	0.8	0.0
CO	0.4	-1.1	0.0	0.4
Dist	-0.2	0.3	0.3	0.0
CO	0.2	-0.1	0.0	0.2
Dist	-0.1	0.0	0.0	0.0
M	120.2	-175.5	125.6	-66.6
Midspan M	112.8		31.9	



(4) Loading pattern for negative design moment at support A*

Loading (5) First and second spans loaded with 3/4 factored live load							
FEM	176.1	-176.1	176.1	-176.1	86.5	-86.5	 (5) Loading pattern for negative design moment at support B*
Dist	-68.1	0.0	0.0	27.4	27.4	33.4	
CO	0.0	-34.6	13.9	0.0	17.0	13.9	
Dist	0.0	6.3	6.3	-5.2	-5.2	-5.4	
CO	3.2	0.0	-2.6	3.2	-2.7	-2.6	
Dist	-1.2	0.8	0.8	-0.2	-0.2	1.0	
CO	0.4	-0.6	-0.1	0.4	0.5	-0.1	
Dist	-0.2	0.2	0.2	-0.3	-0.3	0.0	
CO	0.1	-0.1	-0.1	0.1	0.0	-0.1	
Dist	0.0	0.1	0.1	0.0	0.0	0.1	
CO	0.0	0.0	0.0	0.0	0.0	0.0	
Dist	0.0	0.0	0.0	0.0	0.0	0.0	
M	77.6	-146.0	139.1	-105.7	84.1	-29.5	
Midspan M	74.3		63.7		28.3		

						
Max M^-	131.1	-232.8	213.5	-213.5	232.8	-131.1
Max M^+	123.0		98.0		123.0	

2.4. Design moments

Positive and negative factored moments for the slab system in the direction of analysis are plotted in Figure 9. The negative moments used for design are taken at the faces of supports (rectangle section or equivalent rectangle for circular or polygon sections) but not at distances greater than $0.175\ell_1$ from the centers of supports. ***CSA A23.3-14 (13.8.5.1)***

$$450 \text{ mm} < 0.175 \times 5{,}500 = 926.5 \text{ mm (use face of support location)}$$

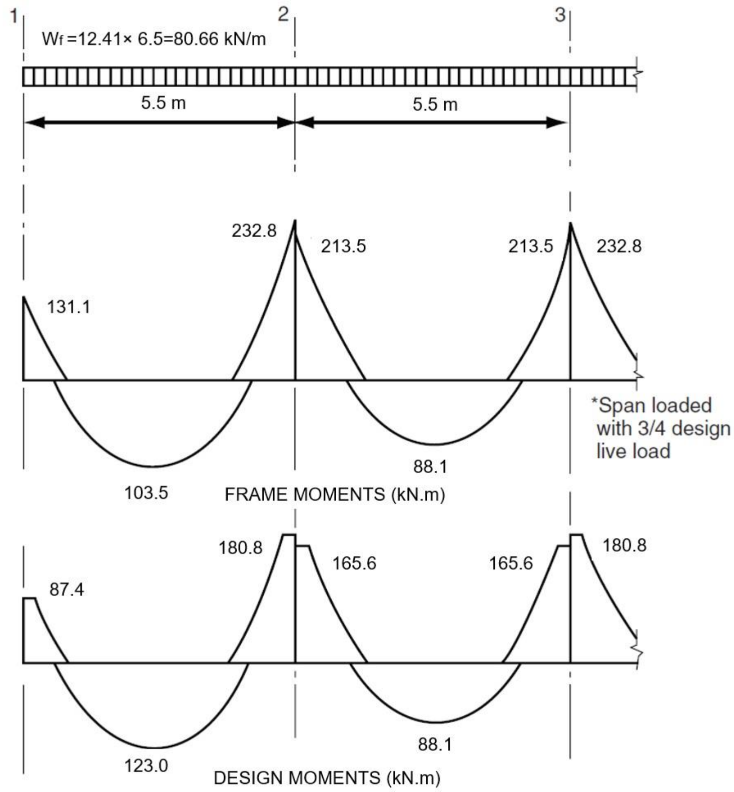


Figure 9 – Positive and Negative Design Moments for Slab-Beam (All Spans Loaded with Full Factored Live Load except as Noted)

2.5. Distribution of design moments

Check Applicability of Direct Design Method:

1. There shall be a minimum of three continuous spans in each direction (3 spans) ***CSA A23.3-14 (13.9.1.2)***
2. Successive span lengths centre-to-centre of supports in each direction shall not differ by more than one-third of the longer span (span lengths are equal) ***CSA A23.3-14 (13.9.1.3)***
3. All loads shall be due to gravity only and uniformly distributed over an entire panel (Loads are uniformly distributed over the entire panel) ***CSA A23.3-14 (13.9.1.4)***
4. The factored live load shall not exceed twice the factored dead load (Factored live-to-dead load ratio of 1.38 < 2.0) ***CSA A23.3-14 (13.9.1.4)***
5. For slabs with beams between supports, the relative effective stiffness of beams in the two directions $(\alpha_1 l_2^2 / \alpha_2 l_1^2)$ is not less than 0.2 or greater than 5.0. ***CSA A23.3-14 (13.9.1.1)***

$\alpha_1 = 3.68$, $l_2 = 5.5 \text{ m} = 5,500 \text{ mm}$

$\alpha_2 = 11.41$, $l_1 = 5.5 \text{m} = 5,500 \text{ mm}$

$$\frac{\alpha_1 l_2^2}{\alpha_2 l_1^2} = \frac{3.12 \times 6,500^2}{9.65 \times 5,500^2} = 0.45 \rightarrow 0.2 < 0.45 < 5.0 \qquad \textbf{\textit{O.K.}}$$

Since all the criteria are met, Direct Design Method can be utilized.

b. Distribute factored moments to column and middle strips:

The negative and positive factored moments at critical sections may be distributed to the column strip and the two half-middle strips of the slab-beam according to the Direct Design Method (DDM) in 13.9, provided that limitations in ***13.9.1.1*** is satisfied. ***CSA A.23.3-14 (13.2)***

Beams shall be reinforced to resist the following fraction of the positive or interior negative factored moments determined by analysis or determined as specified in Clause 13.9.3. ***CSA A.23.3-14 (13.12.2.1)***

Portion of design moment resisted by beam:

$$\frac{\alpha_1}{0.3+\alpha_1}\left(1-\frac{l_2}{3l_1}\right)=\frac{3.12}{0.3+3.12}\left(1-\frac{6.5}{3\times 5.5}\right)=0.553$$

Factored moments at critical sections are summarized in Table 2.

Table 2 - Lateral distribution of factored moments

		Factored Moments (kN.m)	Column Strip				Moments in Two Half-Middle Strips* (kN.m)
			Beam Strip Percent	Beam Strip Moment (kN.m)	Column Strip Percent	Column Strip Moment (kN.m)	
End Span	Exterior Negative	87.39	100	87.39	0.00	0.00	0.00
	Positive	123.05	55.3	68.03	17.4	21.47	33.55
	Interior Negative	180.80	55.3	99.96	17.4	31.55	49.29
Interior Span	Negative	165.61	55.3	91.56	17.4	28.90	45.15
	Positive	97.98	55.3	54.17	17.4	17.10	26.71

*That portion of the factored moment not resisted by the column strip is assigned to the two half-middle strips

2.6. Flexural reinforcement requirements

a. Determine flexural reinforcement required for strip moments

The flexural reinforcement calculation for the column strip of end span – interior negative location is provided below:

$M_f = 31.55$ kN.m

Column strip width, b = (5,500 /2) - 350 = 2,400 mm

Use d_{avg} = 127 mm

In this example, jd is assumed equal to $0.98d$. The assumption will be verified once the area of steel in finalized.

Assume $jd = 0.98\times d = 447.3$ mm

Column strip width, b = (5,500 /2) - 350 = 2,400 mm

Middle strip width, $b = 6,500-2,400-350 = 3,750$ mm

$$A_s = \frac{M_f}{\varphi_s f_y jd} = \frac{31.55\times10^6}{0.85\times400\times447.3} = 207.5 \text{ mm}^2$$

$\alpha_1 = 0.85 - 0.0015 f_c^{'} = 0.81 > 0.67$ ***CSA A23.3-14 (10.1.7)***

Recalculate $'a'$ for the actual $A_s = 207.5 \text{ mm}^2 \rightarrow a = \dfrac{\phi_s A_s f_y}{\phi_c \alpha_1 f'_c b} = \dfrac{0.85\times207.5\times400}{0.65\times0.81\times35\times2,400} = 15.26 \text{ mm}$

$$c = \frac{a}{\beta_1} = \frac{15.26}{0.91} = 16.8 \text{ mm}$$

The tension reinforcement in flexural members shall not be assumed to reach yield unless:

$\dfrac{c}{d} \le \dfrac{700}{700 + f_y}$ ***CSA A23.3-14 (10.5.2)***

$$\frac{16.8}{127} = 0.13 \le 0.64$$

$$jd = d - a/2 = 0.98d$$

$A_{s,\min} = 0.002\times2400\times155 = 744 \text{ mm}^2 > 207.5 \text{ mm}^2$ ***CSA A23.3-14 (7.8.1)***

$\therefore A_S = 774 \text{ mm}^2$

Maximum spacing: ***CSA A23.3-14 (13.10.4)***

- Negative reinforcement in the band defined by b_b: $1.5h_s = 232.5 \text{ mm} \le 250 \text{ mm}$
- Remaining negative moment reinforcement: $3h_s = 465 \text{ mm} \le 500 \text{ mm}$

Provide 6 – 15M bars with A_s = 200 mm^2 and s = 2,400/6 = 400 mm ≤ s_{max}

The flexural reinforcement calculation for the beam strip of end span – interior negative location is provided below:

$M_f = 99.96$ kN.m

Beam strip width, b = 350 mm

Use d = 468 mm

jd is assumed equal to *0.948d*. The assumption will be verified once the area of steel in finalized.

Assume $jd = 0.948\times d = 443.6$ mm

$$A_s = \frac{M_f}{\varphi_s f_y jd} = \frac{99.96\times10^6}{0.85\times400\times443.6} = 662.6 \text{ mm}^2$$

$\alpha_1 = 0.85 - 0.0015 f_c^{'} = 0.81 > 0.67$ ***CSA A23.3-14 (10.1.7)***

$\beta_1 = 0.97 - 0.0025 f_c^{'} = 0.91 > 0.67$ ***CSA A23.3-14 (10.1.7)***

Recalculate $'a'$ for the actual $A_s = 662.6 \text{ mm}^2 \rightarrow a = \dfrac{\phi_s A_s f_y}{\phi_c \alpha_1 f'_c b} = \dfrac{0.85\times662.6\times400}{0.65\times0.81\times35\times350} = 48.75 \text{ mm}$

$$c = \frac{a}{\beta_1} = \frac{48.75}{0.91} = 53.7 \text{ mm}$$

The tension reinforcement in flexural members shall not be assumed to reach yield unless:

$\dfrac{c}{d} \le \dfrac{700}{700 + f_y}$ ***CSA A23.3-14 (10.5.2)***

$$\frac{48.75}{472} = 0.115 \le 0.64$$

$$jd = d - a/2 = 0.948d$$

$$A_{s,\min} = \frac{0.2 \times \sqrt{f_c'}}{f_y} \times b_t \times h = \frac{0.2\sqrt{25}}{400} \times 350 \times 500 = 437.5 \text{ mm}^2$$ **CSA A23.3-14 (10.5.1.2)**

$$\therefore A_s = 662.6 \text{ mm}^2$$

Provide 2 – 25M bars with A_s = 500 mm^2

All the values on Table 3 are calculated based on the procedure outlined above.

Table 3 - Required Slab Reinforcement for Flexure [Elastic Frame Method (EFM)]

Span Location		M_f (kN.m)	b * (mm)	d ** (mm)	A_s Req'd for flexure (mm^2)	Min A_s† (mm^2)	Reinforcement Provided	A_s Prov. for flexure (mm^2)
End Span								
Beam Strip	Exterior Negative	87.39	350	468	575.1	437.5	2 – 25M	1,000
	Positive	68.03	350	458	443.5	437.5	2 – 25M	1,000
	Interior Negative	99.96	350	468	662.6	437.5	2 – 25M	1,000
Column Strip	Exterior Negative	0.00	2,400	127	0.0	744	6 – 15M	1,200
	Positive	21.47	2,400	127	135.3	744	6 – 15M	1,200
	Interior Negative	31.55	2,400	127	200.0	744	6 – 15M	1,200
Middle Strip	Exterior Negative	0.00	3,750	127	0.0	1,162.5	9 – 15M	1,800
	Positive	33.55	3,750	127	212.9	1,162.5	9 – 15M	1,800
	Interior Negative	49.29	3,750	127	316.0	1,162.5	9 – 15M	1,800
Interior Span								
Beam Strip	Positive	54.17	350	457	437.5	437.5	2 – 25M	1,000
Column Strip	Positive	17.10	2,400	127	107.5	744	6 – 15M	1,200
Middle Strip	Positive	26.71	3,750	127	168.8	1,162.5	9 – 15M	1,800

* Column strip width, b = (5,500/2) - 350 = 2,400 mm

* Middle strip width, b = 6,500-2,400-350 = 3,750 mm

* Beam strip width, b = 350 mm

** Use average d = 155 – 20 – 7 = 127 mm for Column and Middle strips

** Use average d = 500 - 30 -13 = 457 mm for Beam strip Positive moment regions

** Use average d = 500 - 20 - 12 = 468 mm for Beam strip Negative moment regions

† Min. A_s = 0.002 × b × h = 0.31 × b for Column and Middle strips **CSA A23.3-14 (7.8.1)**

† Min. A_s = (0.2(f_c')^0.5/f_y*b*d for Beam strip **CSA A23.3-14 (10.5.1.2)**

b. Calculate additional slab reinforcement at columns for moment transfer between slab and column by flexure

Portion of the unbalanced moment transferred by flexure is $\gamma_f \times M_f$

Where:

$$\gamma_f = \frac{1}{1+(2/3)\times\sqrt{b_1/b_2}}$$ **CSA A23.3-14 (13.10.2)**

$b_1 =$ Width width of the critical section for shear measured in the direction of the span for which moments are determined according to CSA A23.3-14, clause 13 (see Figure 10).

$b_2 =$ Width of the critical section for shear measured in the direction perpendicular to b1 according to CSA A23.3-14, clause 13 (see Figure 10).

b_b = Effective slab width = $c_2 + 3\times h_s$ **CSA A23.3-14 (3.2)**

For Exterior Column:

$$b_1 = c_1 + \frac{d}{2} = 450 + \frac{127}{2} = 513.5 \text{ mm}\ ,\ b_2 = c_2 + d = 450 + 127 = 577 \text{ mm}\ ,\ b_b = c_2 + 3\,h = 450 + 3\,(155) = 915 \text{ mm}$$

$$\gamma_f = \frac{1}{1+(2/3)\times\sqrt{513.5/577}} = 0.614$$

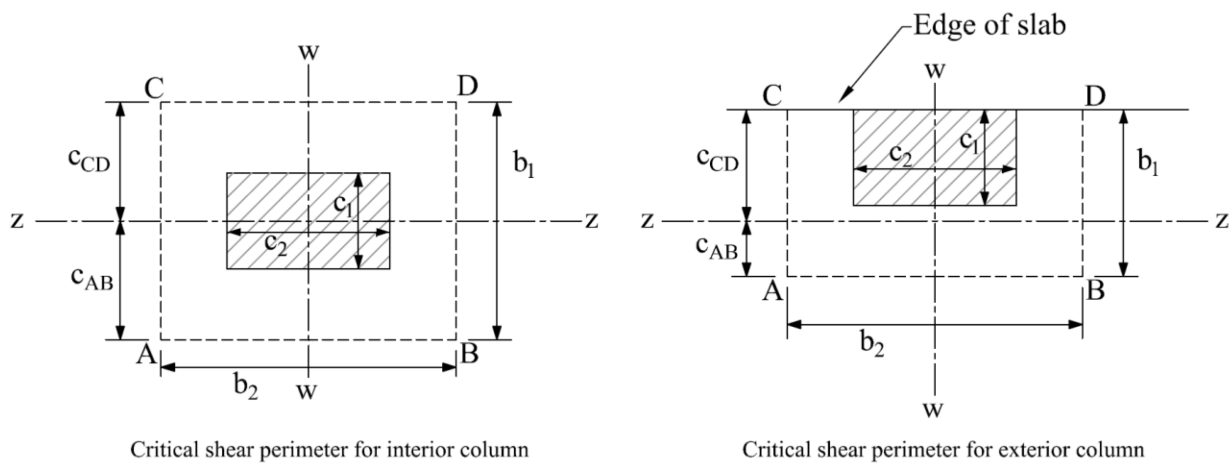


Figure 10 – Critical Shear Perimeters for Columns

$$\gamma_f M_{f,net} = 0.614\times 131.1 = 80.48 \text{ kN.m}$$

$$A_{s,req'd} = \frac{\varphi_c \times 0.81\times f_c' \times b_b}{\varphi_s \times f_y}\left(d - \sqrt{d^2 - \frac{2\times\gamma_f M_{f,net}}{\varphi_c \times 0.81\times f_c' \times b_b}}\right)$$

$$A_{s,req'd} = \frac{0.65\times 0.81\times 25\times 915}{0.85\times 400}\left(117 - \sqrt{117^2 - \frac{2\times 80.48\times 10^6}{0.65\times 0.81\times 25\times 915}}\right) = 3{,}507 \text{ mm}^2$$

$$A_{s,\min} = 0.002\times 2400\times 155 = 744 \text{ mm}^2 < 3{,}507 \text{ mm}^2$$ **CSA A23.3-14 (7.8.1)**

$$\therefore A_{s,req'd} = 3{,}507 \text{ mm}^2$$

$$A_{s,provided} = (A_{s,provided})_{(beam)} + (A_{s,provided})_{(b_b - b_{beam})}$$

$$A_{s,provided} = 2\times 500 + 6\times 200\times \frac{915-350}{2,400} = 1283 \text{ mm}^2 < A_{s,req'd} = 3,507 \text{ mm}^2$$

$\therefore$ Additional slab reinforcement at the exterior column is required.

$$A_{req'd,\,add} = 3507 - 1283 = 2224.5 \text{ mm}^2$$

Use 12 - 15M $\rightarrow A_{provided,\,add} = 12\times 200 = 2,400 \text{ in.}^2 > A_{req'd,\,add} = 2,224.5 \text{ mm}^2$

Table 4 - Additional Slab Reinforcement at columns for moment transfer between slab and column [Elastic Frame Method (EFM)]									
Span Location		**Effective slab width, b_b (mm)**	**d (mm)**	**γ_f**	**M_u^* (kN.m)**	**$\gamma_f M_u$ (kN.m)**	**A_s req'd within b_b (mm^2)**	**A_s prov. for flexure within b_b (mm^2)**	**Add'l Reinf.**
End Span									
Column Strip	Exterior Negative	915	117	0.614	131.1	80.48	3,507	1,283	12-15M
	Interior Negative	915	117	0.600	59.4	35.64	1,022	1,283	-
*M_f is taken at the centerline of the support in Elastic Frame Method solution.									

b. Determine transverse reinforcement required for beam strip shear

The transverse reinforcement calculation for the beam strip of end span – exterior location is provided below.

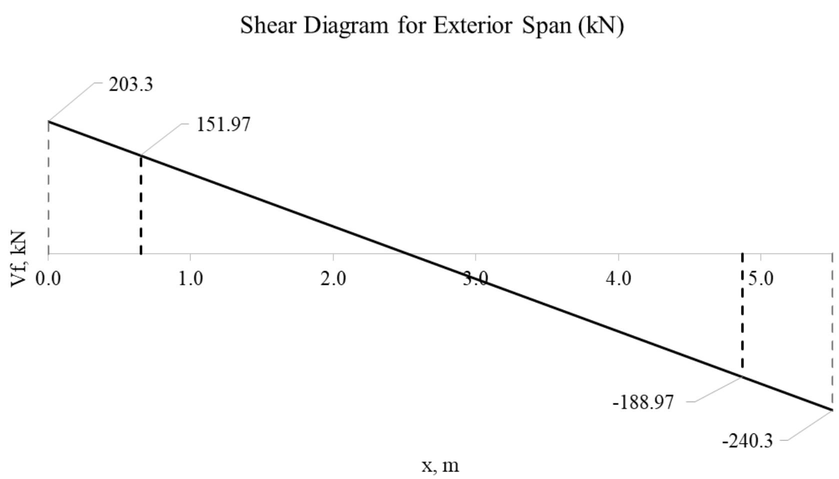


Figure 11 – Shear at critical sections for the end span (at distance d_v from the face of the column)

$d_v = Max\ (0.9d, 0.72h) = Max\ (0.9\times 457, 0.72\times 500) = 411.7\,\text{mm}$ **CSA A23.3-14 (3.2)**

The required shear at a distance d from the face of the supporting column V_{u_d}= 152 kN (Figure 11).

$V_{r,\max} = 0.25\times 0.65\times 25\times 350\times 411.7/1000 = 585.5 \text{ kN} \rightarrow \therefore$ section is adequate **CSA A23.3-14 (11.3.3)**

$V_c = \varphi_c \lambda \beta \sqrt{f_c'}\, b_w d_v$ **CSA A23.3-14 (Eq. 11.5)**

$V_c = 0.65\times 1\times 0.18\times\sqrt{25}\times 350\times 411.7/1,000 = 84.21\text{ kN}<152\text{ kN}$ ∴ Stirrups are required.

Distance from the column face beyond which minimum reinforcement is required:

$V_s = V_{f\_d} - V_c$ ***ACI 318-14 (22.5.10.1)***

$V_s = 152 - 84.21 = 67.8\text{ kN}$

$$\left(\frac{A_v}{s}\right)_{req} = \frac{V_f - V_c}{\phi\times f_{yt}\times d_v\times\cot\theta} = \frac{67.8\times 1000}{0.85\times 400\times 411.7\times\cot 35^\circ} = 0.338\text{ mm}^2/\text{mm}$$ ***CSA A23.3-14 (11.3.5.1)***

Where $\theta = 35^\circ$ ***CSA A23.3-14 (11.3.6.2)***

$$\left(\frac{A_v}{s}\right)_{\min} = \frac{0.06\times\sqrt{f_c^{'}}\times b_w}{f_{yt}}$$ ***CSA A23.3-14 (11.2.8.2)***

$$\left(\frac{A_v}{s}\right)_{\min} = \frac{0.06\times\sqrt{25}\times 350}{400} = 0.263\text{ mm}^2/\text{mm}$$

$$s_{req} = \frac{A_v}{\left(\frac{A_v}{s}\right)_{req}} = \frac{2\times 100}{0.263} = 590.9\text{ mm}$$

Check whether the required spacing based on the shear demand meets the spacing limits for shear reinforcement per ***CSA A23.3-14 (11.3.8)***.

$0.125\lambda\varphi_c f_c^{'} b_w d_v = 292.73 > V_f$ ***CSA A23.3-14 (11.3.8.3)***

Therefore, maximum stirrup spacing shall be the smallest of $0.7d_v$ and *600 mm.* ***CSA A23.3-14 (11.3.8.1)***

$$s_{\max} = \textit{lesser of}\begin{bmatrix}0.7d_v\\ 600\text{ mm}\end{bmatrix} = \textit{lesser of}\begin{bmatrix}0.7\times 411.7\\ 600\text{ mm}\end{bmatrix} = \textit{lesser of}\begin{bmatrix}288\text{ mm}\\ 600\text{ mm}\end{bmatrix} = 288\text{ mm}$$

Since $s_{req'd} > s_{max} \rightarrow$ use s_{max}

Select $s_{provided}$ = 280 mm – 10M stirrups with first stirrup located at distance 140 mm from the column face.

The distance where the shear is zero is calculated as follows:

$$x = \frac{l}{V_{f,L} + V_{f,R}}\times V_{u,L} = \frac{5.5}{203.3 + 240.3}\times 203.3 = 2.52\text{ m} = 2{,}520\text{ mm}$$

The distance at which no shear reinforcement is required is calculated as follows:

$$x_1 = x - \frac{x}{V_f}\times V_c = 2.52 - \frac{2.52}{203.3}\times 84.21 = 1.48\text{ m} = 1{,}480\text{ mm}$$

$$\#\ of\ stirrups = \frac{x_1 - \frac{c_1}{2} - \frac{s_{provided}}{2}}{s_{provided}} + 1 = \frac{1,480 - \frac{450}{2} - \frac{280}{2}}{280} + 1 \approx 6 \rightarrow \text{use 6 stirrups}$$

All the values on Table 5 are calculated based on the procedure outlined above.

Table 5 - Required Beam Reinforcement for Shear					
Span Location	**$A_{v,min}/s$ mm²/mm**	**$A_{v,req'd}/s$ mm²/mm**	**$s_{req'd}$ mm**	**s_{max} mm**	**Reinforcement Provided**
End Span					
Exterior	0.263	0.338	590	288	6 – 10M @ 280 mm
Interior	0.263	0.535	373	288	6 – 10M @ 280 mm
Interior Span					
Interior	0.263	0.431	464	288	8 – 10M @ 280 mm

2.7. Column design moments

The unbalanced moment from the slab-beams at the supports of the frame are distributed to the actual columns above and below the slab-beam in proportion to the relative stiffness of the actual columns. Referring to Fig. 9, the unbalanced moment at joints 1 and 2 are:

Joint 1 = +131.1 kN.m

Joint 2 = -204.0 + 194.6 = -9.45 kN.m

The stiffness and carry-over factors of the actual columns and the distribution of the unbalanced moments to the exterior and interior columns are shown in Fig 12.

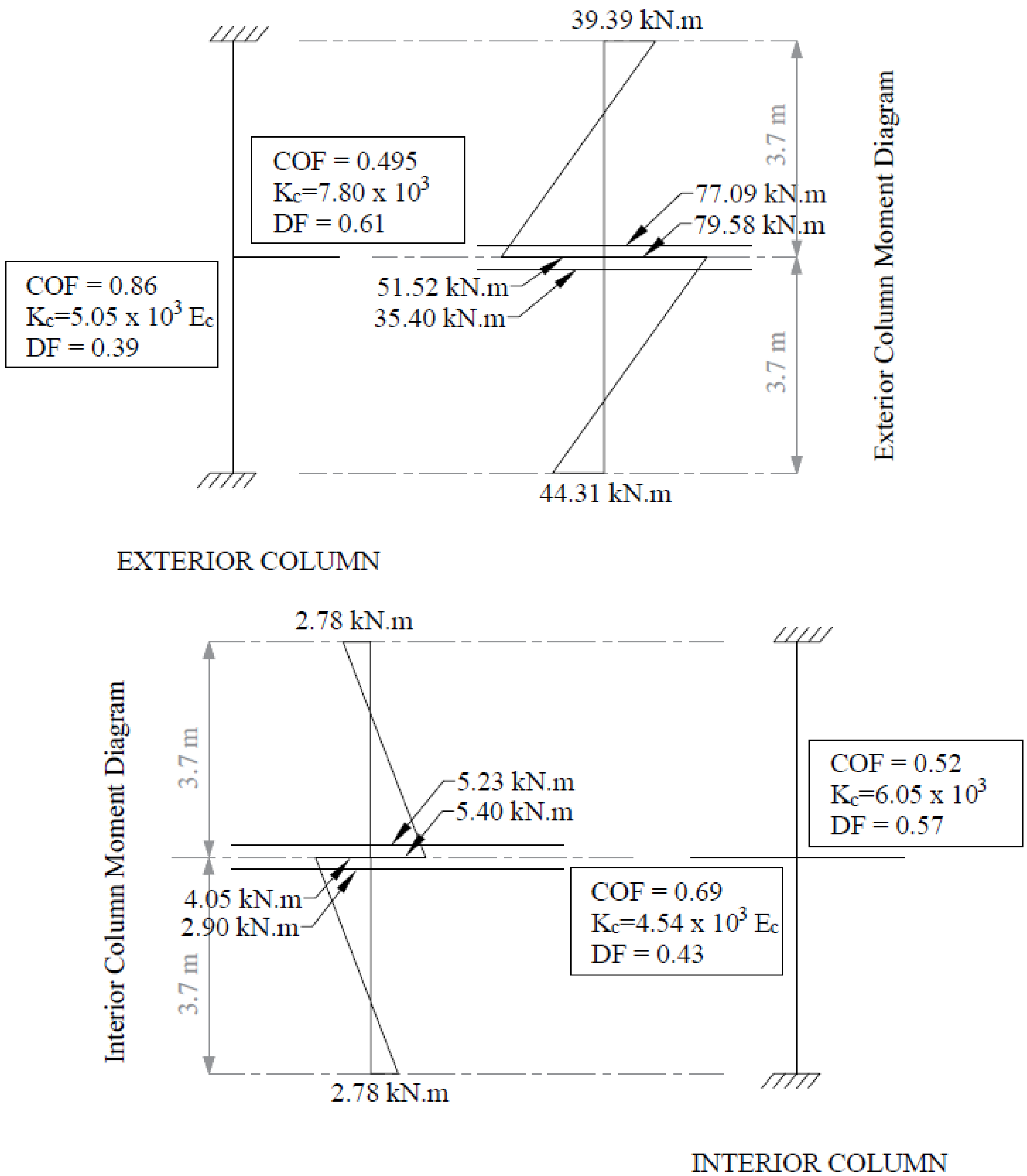


Figure 12 - Column Moments (Unbalanced Moments from Slab-Beam)

In summary:

Design moment in exterior column = 59.57 kN.m

Design moment in interior column = 5.40 kN.m

The moments determined above are combined with the factored axial loads (for each story) and factored moments in the transverse direction for design of column sections. A detailed analysis to obtain the moment values at the face of interior, exterior, and corner columns from the unbalanced moment values can be found in the "Two-Way Flat Plate Concrete Floor Slab Design" example.

3. Design of Interior, Edge, and Corner Columns

The design of interior, edge, and corner columns is explained in the "Two-Way Flat Plate Concrete Floor Slab Design" example.

4. Two-Way Slab Shear Strength

Shear strength of the slab in the vicinity of columns/supports includes an evaluation of one-way shear (beam action) and two-way shear (punching) in accordance with CSA A23.3-14 clause 13.

4.1. One-Way (Beam action) Shear Strength

One-way shear is critical at a distance d_v from the face of the column. Figure 13 shows the V_f at the critical sections around each column. Since there is no shear reinforcement, the design shear capacity of the section equals to the design shear capacity of the concrete:

$$V_r = V_c + V_s + V_p = V_c \quad , \quad (V_s = V_p = 0)$$ ***CSA A23.3-14 (Eq. 11.4)***

Where:

$$V_c = \varphi_c \lambda \beta \sqrt{f_c^{'}} b_w d_v$$ ***CSA A23.3-14 (Eq. 11.5)***

$\lambda = 1$ for normal weight concrete

$\beta = 0.21$ for slabs with overall thickness not greater than 350 mm ***CSA A23.3-14 (11.3.6.2)***

$d_v = \text{Max}\ (0.9 d_{avg}, 0.72h) = \text{Max}\ (0.9\times127, 0.72\times155) = 114$ mm ***CSA A23.3-14 (3.2)***

$\sqrt{f_c^{'}} = 5$ MPa < 8 MPa ***CSA A23.3-14 (11.3.4)***

$$V_c = 0.65\times1\times0.21\times\sqrt{25}\times5,500\times\frac{114}{1000} = 427.92 \text{ kN} > V_f$$

Because $V_r \geq V_f$ at all the critical sections, the slab has adequate one-way shear strength.

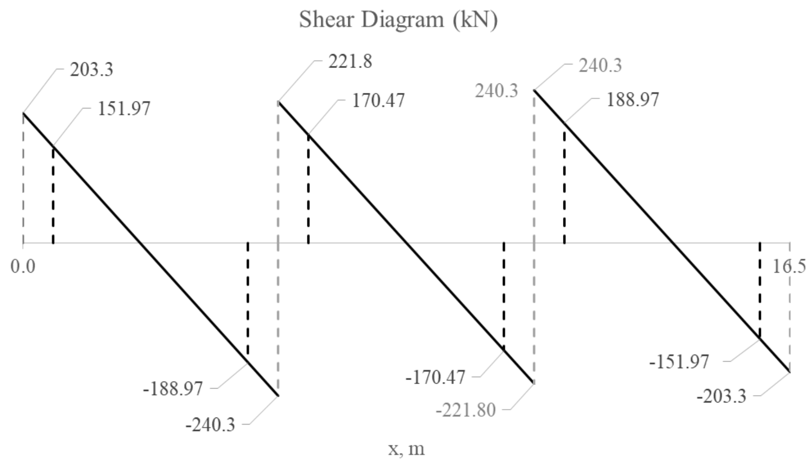


Figure 13 – One-way shear at critical sections (at distance d_v from the face of the supporting column)

4.2. Two-Way (Punching) Shear Strength

Two-way shear is critical on a rectangular section located at $d_{slab}/2$ away from the face of the column. The factored shear force V_f in the critical section is calculated as the reaction at the centroid of the critical section minus the self-weight and any superimposed surface dead and live load acting within the critical section.

The factored unbalanced moment used for shear transfer, M_{unb}, is calculated as the sum of the joint moments to the left and right. Moment of the vertical reaction with respect to the centroid of the critical section is also taken into account.

For the exterior column:

$$V_f = 203.2 - 12.41\left(\frac{514 \times 578}{10^6}\right) = 199.5 \text{ kN}$$

$$M_{unb} = 93.1 - 43.56\left(\frac{20.5 - 9.09 - 18/2}{12}\right) = 84.37 \text{ ft-kip}$$

For the exterior column in Figure 14, the location of the centroidal axis z-z is:

$$c_{AB} = \frac{\text{moment of area of the sides about AB}}{\text{area of the sides}}$$

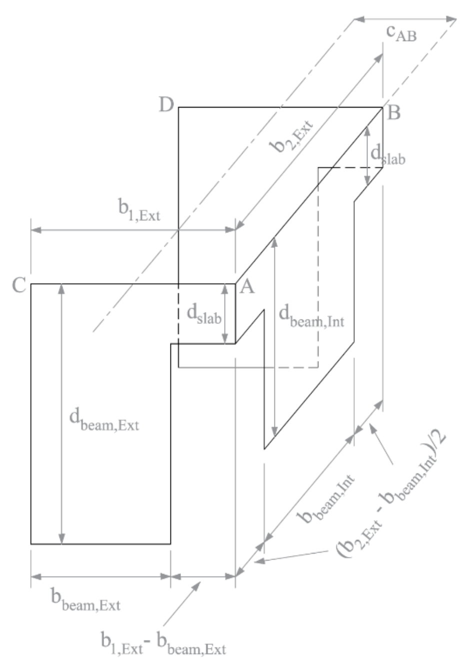


Figure 14 – Critical section of exterior support of interior frame

$$c_{AB} = \frac{2(350 \times 672 \times (514 - 350/2) + ((514-350) \times 127 \times (514-350)/2)}{2 \times (350 \times 672 + (514-350) \times 127) + 350 \times 472 + (577-514) \times 127} = 230.4 \text{ mm}$$

$$A_c = 2\times(350\times672+127\times(514-350))+127\times(577-350)+350\times472 = 7.05\times10^5 \text{ mm}^2$$

The polar moment J_c of the shear perimeter is:

$$J_c = 2\left[\frac{b_{beam,Ext}d^3_{beam,Ext}}{12}+\frac{d_{beam,Ext}b_{beam,Ext}{}^3}{12}+\left[b_{beam,Ext}d_{beam,Ext}\right]\left[\frac{b_{beam,Ext}}{2}+\left(b_1-b_{beam,Ext}\right)-c_{AB}\right]^2\right]$$

$$+2\left[\frac{\left(b_1-b_{beam,Ext}\right)d^3_{slab,Ext}}{12}+\frac{d_{slab}\left(b_1-b_{beam,Ext}\right)^3}{12}+\left[\left(b_1-b_{beam,Ext}\right)d_{slab}\right]\left[c_{AB}-\frac{b_1-b_{beam,Ext}}{2}\right]^2\right]$$

$$+\left[b_{beam,Int}\ d_{beam.Int}+\left(b_2-b_{beam,Int}\right)\ d_{slab}\right]\ c^2_{AB}$$

$$J_c = 2\left[\frac{350\times672^3}{12}+\frac{672\times350^3}{12}+\left[350\times672\right]\left[\frac{350}{2}+\left(514-350\right)-230.4\right]^2\right]$$

$$+2\left[\frac{\left(514-350\right)\times127^3}{12}+\frac{127\times\left(514-350\right)^3}{12}+\left[\left(514-350\right)\times127\right]\left[230.4-\frac{514-350}{2}\right]^2\right]$$

$$+\left[350\times457+\left(577-350\right)\ \times127\right]\ \times230.4^2$$

$$J_c = 3.94\times10^{10}\text{ mm}^4$$

$$\gamma_v = 1-\gamma_f = 1-0.614 = 0.386$$ ***CSA A23.3-14 (Eq. 13.8)***

The length of the critical perimeter for the exterior column:

$$b_o = 2\times(450+127/2)+(450+127) = 1604 \text{ mm}$$

$$v_f = \frac{V_f}{b_o\times d}+\frac{\gamma_v M_{unb} e}{J}$$ ***CSA A23.3-14 (Eq.13.9)***

$$v_f = \frac{199.5\times1000}{7.05\times10^5}+\frac{0.386\times43.7\times1000\times230.4}{3.94\times10^{10}} = 0.538 \text{ MPa}$$

The factored resisting shear stress, V_r shall be the smallest of: ***CSA A23.3-14 (13.3.4.1)***

a) $$v_r = v_c = \left(1+\frac{2}{\beta_c}\right)0.19\lambda\phi_c\sqrt{f'_c} = \left(1+\frac{2}{1}\right)0.19\times0.65\times\sqrt{25} = 1.85 \text{ MPa}$$

b) $$v_r = v_c = \left(\frac{\alpha_s d}{b_o}+0.19\right)\lambda\phi_c\sqrt{f'_c} = \left(\frac{3\times127}{1604}+0.19\right)\times1\times0.65\times\sqrt{25} = 1.39 \text{ MPa}$$

c) $$v_r = v_c = 0.38\lambda\phi_c\sqrt{f'_c} = 0.38\times1\times0.65\times\sqrt{25} = 1.24 \text{ MPa}$$

In this example, since the d_{avg} = 440.1 mm around the joint for two-way shear, exceeds 300 mm, therefore the value of v_c obtained above shall be multiplied by 1300/(1000+d). ***CSA A23.3-14 (13.3.4.3)***

$$\mathrm{v_c} = \frac{1300}{(1000+d)} \times 1.24 = \frac{1300}{(1000+440.1)} \times 1.24 = 1.115 \text{ MPa}$$

Since $v_r \geq v_f$ at the critical section, the slab has adequate two-way shear strength at this joint.

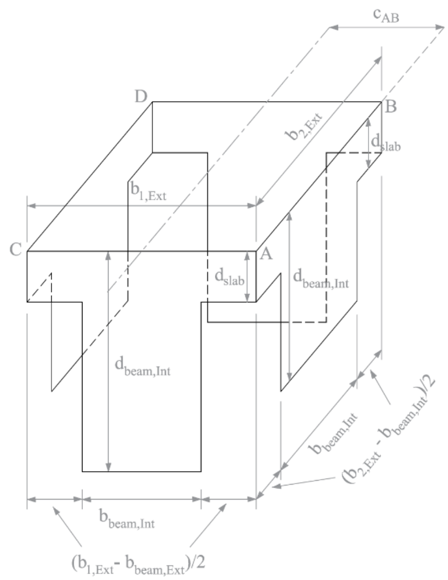


Figure 15 – Critical section of interior support of interior frame

For the interior column:

$$\mathrm{V_f} = 240.3 + 221.8 - 12.41\left(\frac{577 \times 577}{10^6}\right) = 458 \text{ kN}$$

$$\mathrm{M_{unb}} = 232.8 - 213.8 - 458(0) = 19.0 \text{ kN.m}$$

For the interior column in Figure 15, the location of the centroidal axis z-z is:

$$\mathrm{c_{AB}} = \frac{b_{1,Int}}{2} = \frac{577}{2} = 288.5 \text{ mm}$$

$$\mathrm{A_c} = 4 \times (350 \times 472 + (577-350) \times 127) = 7.76 \times 10^5 \text{ mm}^2$$

The polar moment $\mathrm{J_c}$ of the shear perimeter is:

$$\mathrm{J}_c = 2\left[\frac{b_{beam,Int} d_{beam,Int}^3}{12} + \frac{d_{beam,Int} b_{beam,Int}^3}{12} + \left[b_{beam,Int} d_{beam,Int}\right]\left[\frac{b_{beam,Int}}{2} + \left(\frac{b_1 - b_{beam,Int}}{2}\right) - c_{AB}\right]^2\right]$$

$$+2\left[\frac{\left(\frac{b_1 - b_{beam,Int}}{2}\right) d_{slab,Int}^3}{12} + \frac{d_{slab}\left(\frac{b_1 - b_{beam,Int}}{2}\right)^3}{12} + \left[\left(\frac{b_1 - b_{beam,Int}}{2}\right) d_{slab}\right]\left[c_{AB} - \frac{b_1 - b_{beam,Int}}{2 \times 2}\right]^2\right]$$

$$+2\left[b_{beam,Int}\ d_{beam.Int} + \left(b_2 - b_{beam,Int}\right)\ d_{slab}\right]\ c_{AB}^2$$

$$\mathrm{J}_c = 2\left[\frac{350 \times 472^3}{12} + \frac{472 \times 350^3}{12} + \left[350 \times 472\right]\left[\frac{350}{2} + \left(\frac{577-350}{2}\right) - 288.5\right]^2\right]$$

$$+2\left[\frac{\left(\frac{577-350}{2}\right) \times 127^3}{12} + \frac{127 \times \left(\frac{577-350}{2}\right)^3}{12} + \left[\left(\frac{577-350}{2}\right) \times 127\right]\left[288.5 - \frac{577-350}{2 \times 2}\right]^2\right]$$

$$+\left[350 \times 472 + \left(577-350\right)\ \times 127\right]\ \times 288.5^2$$

$$J_c = 4.5\times10^{10}\ \text{mm}^4$$

$$\gamma_v = 1-\gamma_f = 1-0.600 = 0.400$$ **ACI 318-14 (Eq. 8.4.4.2.2)**

The length of the critical perimeter for the exterior column:

$$b_o = 4\times(450+127) = 2{,}308\ \text{mm}$$

$$v_f = \frac{V_f}{b_o\times d}+\frac{\gamma_v M_{unb} e}{J}$$ **CSA A23.3-14 (Eq.13.9)**

$$v_f = \frac{458\times1{,}000}{7.76\times10^5}+\frac{0.4\times19.0\times1{,}000\times288.5}{4.5\times10^{10}} = 0.639\ \text{MPa}$$

The factored resisting shear stress, V_r shall be the smallest of: **CSA A23.3-14 (13.3.4.1)**

a) $$v_r = v_c = \left(1+\frac{2}{\beta_c}\right)0.19\lambda\phi_c\sqrt{f'_c} = \left(1+\frac{2}{1}\right)0.19\times0.65\times\sqrt{25} = 1.85\ \text{MPa}$$

b) $$v_r = v_c = \left(\frac{\alpha_s d}{b_o}+0.19\right)\lambda\phi_c\sqrt{f'_c} = \left(\frac{4\times127}{2{,}308}+0.19\right)\times1\times0.65\times\sqrt{25} = 1.33\ \text{MPa}$$

c) $$v_r = v_c = 0.38\lambda\phi_c\sqrt{f'_c} = 0.38\times1\times0.65\times\sqrt{25} = 1.24\ \text{MPa}$$

In this example, since the d_{avg} = 336.3 mm around the joint for two-way shear, exceeds 300 mm, therefore the value of v_c obtained above shall be multiplied by 1300/(1000+d). **CSA A23.3-14 (13.3.4.3)**

$$v_c = \frac{1300}{(1000+d)}\times1.24 = \frac{1300}{(1000+336.3)}\times1.24 = 1.201\ \text{MPa}$$

Since $v_r \geq v_f$ at the critical section, the slab has adequate two-way shear strength at this joint.

5. Two-Way Slab Deflection Control (Serviceability Requirements)

Since the slab thickness was selected based on the minimum slab thickness equations in CSA A23.3-14, the deflection calculations are not required. However, the calculations of immediate and time-dependent deflections are covered in this section for illustration and comparison with spSlab model results.

5.1. Immediate (Instantaneous) Deflections

The calculation of deflections for two-way slabs is challenging even if linear elastic behavior can be assumed. Elastic analysis for three service load levels (D, $D + L_{sustained}$, $D+L_{Full}$) is used to obtain immediate deflections of the two-way slab in this example. However, other procedures may be used if they result in predictions of deflection in reasonable agreement with the results of comprehensive tests. **ACI 318-14 (24.2.3)**

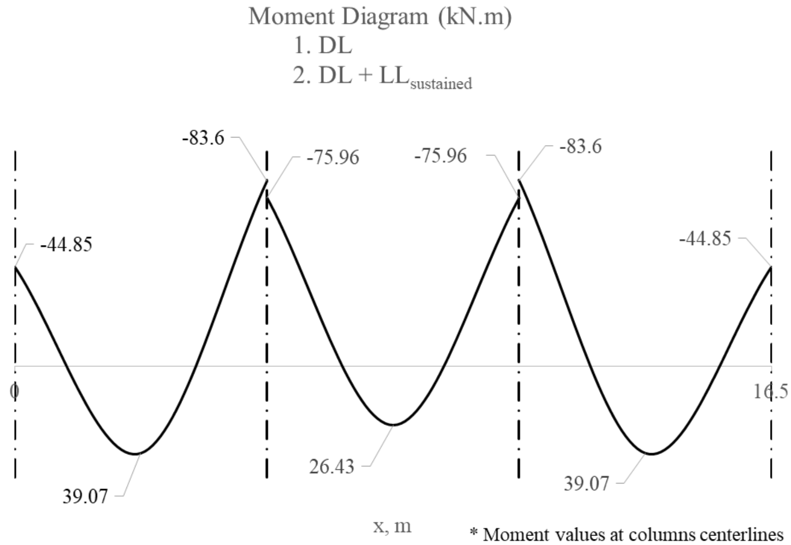
The effective moment of inertia (I_e) is used to account for the cracking effect on the flexural stiffness of the slab. I_e for uncracked section ($M_{cr} > M_a$) is equal to I_g. When the section is cracked ($M_{cr} < M_a$), then the following equation should be used:

$$I_e = I_{cr} + \left(I_g - I_{cr}\right)\left(\frac{M_{cr}}{M_a}\right)^3 \leq I_g$$ **CSA A23.3-14 (Eq.9.1)**

Where:

M_a = Maximum moment in member due to service loads at stage deflection is calculated.

The values of the maximum moments for the three service load levels are calculated from structural analysis as shown previously in this document. These moments are shown in Figure 16.



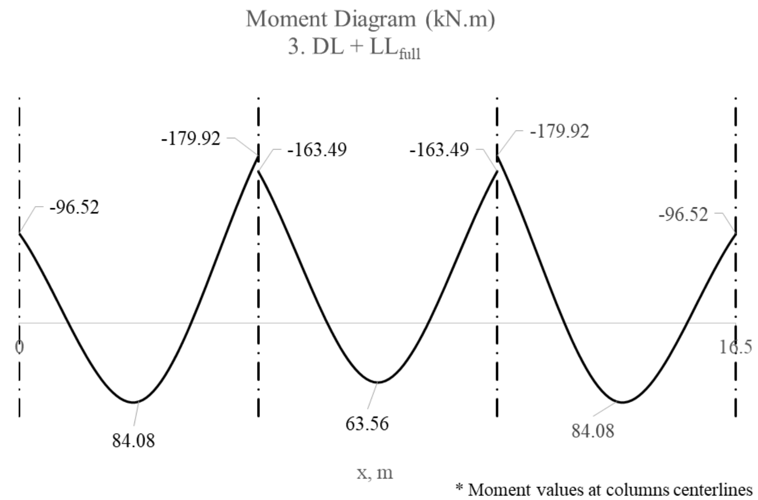


Figure 16 – Maximum Moments for the Three Service Load Levels

For positive moment (midspan) section of the exterior span:

M_{cr} = Cracking moment.

$$M_{cr} = \frac{f_r I_g}{Y_t} = \frac{(3.00/2)\times(9.95\times10^9)}{395.74}\times10^{-6} = 37.73 \text{ kN.m}$$ **CSA A23.3-14 (Eq.9.2)**

f_r should be taken as half of Eq.8.3 ***CSA A23.3-14 (9.8.2.3)***

f_r = Modulus of rapture of concrete.

$$f_r = 0.6\lambda\sqrt{f_c^{'}} = 0.6\times1.0\times\sqrt{25} = 3.00 \text{ MPa}$$ ***CSA A23.3-14 (Eq.8.3)***

I_g = Moment of inertia of the gross uncracked concrete section

$I_g = 9.95\times10^9$ mm^4 for T-section (see Figure 21)

y_t = Distance from centroidal axis of gross section, neglecting reinforcement, to tension face, in.

$y_t = 395.74$ mm (see Figure 17)

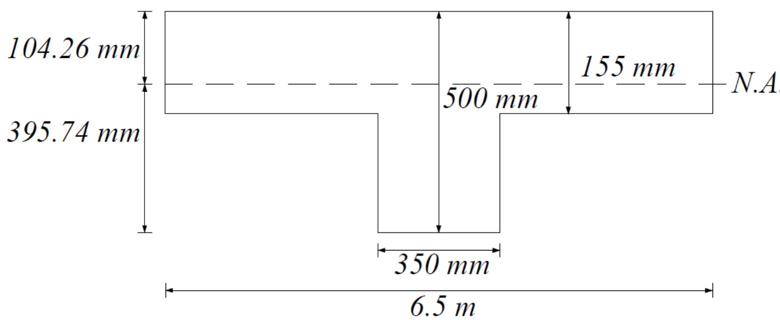


Figure 17 – I_g calculations for slab section near support

I_{cr} = Moment of inertia of the cracked section transformed to concrete.

CAC Concrete Design Handbook 4th Edition (5.2.3)

As calculated previously, the positive reinforcement for the end span frame strip is 15 – 15M bars located at 20 mm along the slab section from the bottom of the slab and 2 – 25M bars located at 30 mm along the beam section from the bottom of the beam. Three of the slab section bars are not continuous and will be excluded from the calculation of I_{cr}. Figure 18 shows all the parameters needed to calculate the moment of inertia of the cracked section transformed to concrete at midspan.

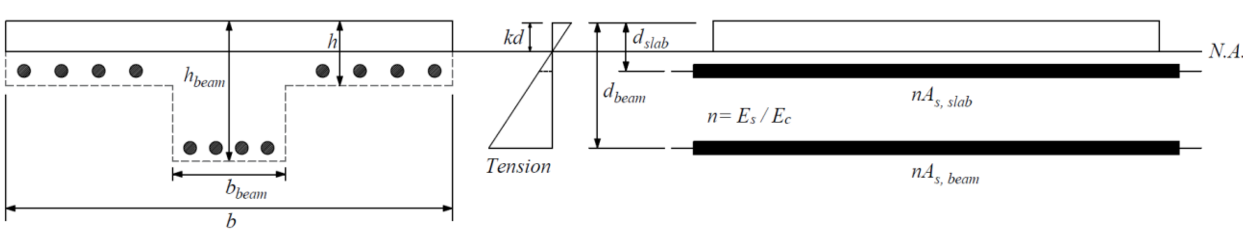


Figure 18 – Cracked Transformed Section (positive moment section)

E_{cs} = Modulus of elasticity of slab concrete.

$$E_{cs} = (3,300\sqrt{f_c^{'}} + 6,900)\left(\frac{\gamma_c}{2,300}\right)^{1.5} = (3,300\sqrt{25} + 6,900)\left(\frac{2,447}{2,300}\right)^{1.5} = 25,684 \text{ MPa}$$ ***CSA A23.3-14(8.6.2.2)***

$$n = \frac{E_s}{E_{cs}} = \frac{200,000}{25,684} = 7.79$$ ***CAC Concrete Design Handbook 4th Edition (Table 6.2a)***

$$a = \frac{b}{2} = \frac{6,500}{2} = 3,250 \text{ mm}$$

$$b = n\,A_{s,beam} + n\,A_{s,slab} = 7.79\times(2\times500)+7.79\times(12\times200) = 26,476.1 \text{ mm}^2$$

$$c = -1\times\left(n\,A_{s,beam}d_{s,beam} + n\,A_{s,slab}d_{s,slab}\right) = -1\times\left(7.79\times(2\times500)\times457+7.79\times(12\times200)\times127\right) = -5.93\times10^{-6} \text{ mm}^3$$

$$kd = \frac{-b\pm\sqrt{b^2-4ac}}{2a} = \frac{-26,476.1\pm\sqrt{26,476.1^2-4\times3,250\times-5.93\times10^6}}{2\times3,250} = 38.84 \text{ mm}$$

$$I_{cr} = \frac{b(kd)^3}{3} + nA_{s,slab}(d_{slab}-kd)^2 + nA_{s,beam}(d_{beam}-kd)^2$$

$$I_{cr} = \frac{6,500\times(38.84)^3}{3} + 7.79\times(12\times200)(127-38.84)^2 + 7.79\times(2\times500)(457-38.84)^2 = 1.63\times10^9 \text{ mm}^4$$

<u>For negative moment section (near the interior support of the end span):</u>

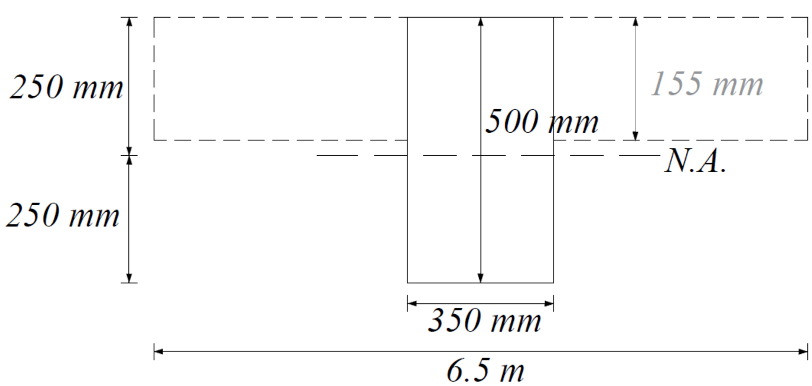
The negative reinforcement for the end span frame strip near the interior support is 27 #4 bars located at 1.0 in. along the section from the top of the slab.

$$M_{cr} = \frac{f_r I_g}{Y_t} = \frac{(3.00/2)\times(3.65\times10^9)}{250}\times10^{-6} = 21.88 \text{ kN.m}$$ ***<u>CSA A23.3-14 (Eq.9.2)</u>***

$$f_r = 0.6\lambda\sqrt{f_c'} = 0.6\times1.0\times\sqrt{25} = 3.00 \text{ MPa}$$ ***<u>CSA A23.3-14 (Eq.8.3)</u>***

$$I_g = 3.65\times10^9 \text{ mm}^4$$

$$y_t = 250 \text{ mm}$$



<u>Figure 19 – I_g calculations for slab section near support</u>

$$E_{cs} = (3,300\sqrt{f_c'}+6,900)\left(\frac{\gamma_c}{2,300}\right)^{1.5} = (3,300\sqrt{25}+6,900)\left(\frac{2,447}{2,300}\right)^{1.5} = 25,684 \text{ MPa}$$ ***<u>CSA A23.3-14(8.6.2.2)</u>***

$$n = \frac{E_s}{E_{cs}} = \frac{200,000}{25,684} = 7.79$$

CAC Concrete Design Handbook 4th Edition (Table 6.2a)

$$B = \frac{b_{beam}}{n\,A_{s,total}} = \frac{350}{7.79\times(15\times200+2\times500)} = 0.011\ \text{mm}^{-1}$$

CAC Concrete Design Handbook 4th Edition (Table 6.2a)

$$kd = \frac{\sqrt{2dB+1}-1}{B} = \frac{\sqrt{2\times468\times0.011+1}-1}{468} = 213\ \text{mm}$$

CAC Concrete Design Handbook 4th Edition (Table 6.2a)

$$I_{cr} = \frac{b_{beam}(kd)^3}{3} + nA_{s,total}(d-kd)^2$$

CAC Concrete Design Handbook 4th Edition (Table 6.2a)

$$I_{cr} = \frac{350\times(213)^3}{3} + 7.79\times(15\times200+2\times500)\times(468-213)^2 = 3.15\times10^9\ \text{mm}^4$$

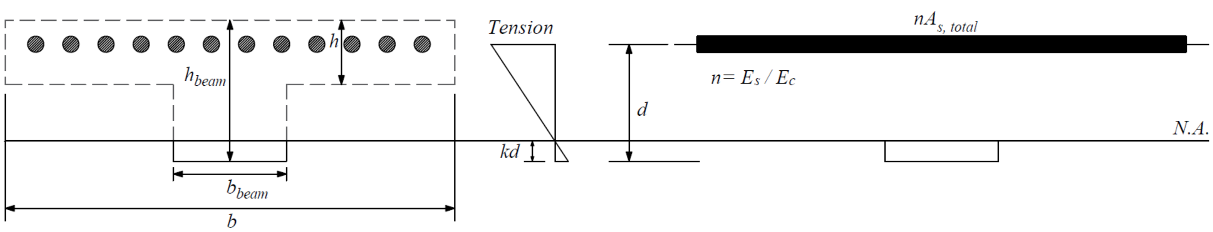


Figure 20 – Cracked Transformed Section (interior negative moment section for end span)

The effective moment of inertia procedure described in the Code is considered sufficiently accurate to estimate deflections. The effective moment of inertia, I_e, was developed to provide a transition between the upper and lower bounds of I_g and I_{cr} as a function of the ratio M_{cr}/M_a. For conventionally reinforced (nonprestressed) members, the effective moment of inertia, I_e, shall be calculated by by Eq. (9.1) in CSA A23.3-14 unless obtained by a more comprehensive analysis.

For continuous prismatic members, the effective moment of inertia may be taken as the weighted average of the values obtained from Eq. (9.1) in CSA A23.3-14 for the critical positive and negative moment sections.

CSA A23.3-14(9.8.2.4)

For the exterior span (span with one end continuous) with service load level ($D+LL_{full}$):

$$I_e^- = I_{cr} + \left(I_g - I_{cr}\right)\left(\frac{M_{cr}}{M_a}\right)^3,\ M_{cr} = 21.88\ \text{kN.m} < M_a = 179.92\ \text{kN.m}$$

ACI 318-14 (24.2.3.5a)

Where I_e^- is the effective moment of inertia for the critical negative moment section (near the support).

$$I_e^- = 3.15\times10^9 + \left(3.65\times10^9 - 3.15\times10^9\right)\left(\frac{21.88}{179.92}\right)^3 = 3.15\times10^9\ \text{mm}^4$$

For positive moment section (midspan):

$$I_e^+ = I_{cr} + \left(I_g - I_{cr}\right)\left(\frac{M_{cr}}{M_a}\right)^3 \text{ , } M_{cr} = 37.73 \text{ kN.m} < M_a = 39.07 \text{ kN.m}$$

Where I_e^+ is the effective moment of inertia for the critical positive moment section (midpan).

$$I_e^+ = 1.63\times10^9 + \left(9.95\times10^9 - 1.63\times10^9\right)\left(\frac{37.73}{84.08}\right)^3 = 2.39\times10^9 \text{ mm}^4$$

Where I_e^+ is the effective moment of inertia for the critical positive moment section (midspan).

Since midspan stiffness (including the effect of cracking) has a dominant effect on deflections, midspan section is heavily represented in calculation of I_e and this is considered satisfactory in approximate deflection calculations. The averaged effective moment of inertia ($I_{e,avg}$) is given by:

$$I_{e,avg} = 0.85\, I_e^+ + 0.15\, I_e^- \text{ for end span}$$ **CSA A23.3-14 (9.8.2.4)**

$$I_{e,avg} = 0.85\,\left(2.39\times10^9\right) + 0.15\,\left(3.15\times10^9\right) = 2.50\times10^9 \text{ mm}^4$$

Where:

I_e^- = The effective moment of inertia for the critical negative moment section near the support.

I_e^+ = The effective moment of inertia for the critical positive moment section (midspan).

For the interior span (span with both ends continuous) with service load level ($D+LL_{full}$):

$$I_e^- = I_{cr} + \left(I_g - I_{cr}\right)\left(\frac{M_{cr}}{M_a}\right)^3 \text{ , } M_{cr} = 21.88 \text{ kN.m} < M_a = 163.49 \text{ kN.m}$$

ACI 318-14 (24.2.3.5a)

$$I_e^- = 3.15\times10^9 + \left(3.65\times10^9 - 3.15\times10^9\right)\left(\frac{21.88}{163.49}\right)^3 = 3.15\times10^9 \text{ mm}^4$$

For positive moment section (midspan):

$$I_e^+ = I_{cr} + \left(I_g - I_{cr}\right)\left(\frac{M_{cr}}{M_a}\right)^3 \text{ , } M_{cr} = 37.73 \text{ kN.m} < M_a = 56.88 \text{ kN.m}$$

Where I_e^+ is the effective moment of inertia for the critical positive moment section (midpan).

$$I_e^+ = 1.63\times10^9 + \left(9.95\times10^9 - 1.63\times10^9\right)\left(\frac{37.73}{56.88}\right)^3 = 4.06\times10^9 \text{ mm}^4$$

The averaged effective moment of inertia ($I_{e,avg}$) is given by:

$$I_{e,avg} = 0.70\, I_e^+ + 0.15\,\left(I_{e,l}^- + I_{e,r}^-\right) \text{ for interior span}$$ **CSA A23.3-14 (9.8.2.4)**

$$I_{e,avg} = 0.70\left(4.06\times10^9\right) + 0.15\left(3.15\times10^9 + 3.15\times10^9\right) = 3.79\times10^9 \text{ mm}^4$$

Where:

$I^{-}_{e,l}$ = The effective moment of inertia for the critical negative moment section near the left support.

$I^{-}_{e,R}$ = The effective moment of inertia for the critical negative moment section near the right support.

Table 6 provides a summary of the required parameters and calculated values needed for deflections for exterior and interior equivalent frame. It also provides a summary of the same values for column strip and middle strip to facilitate calculation of panel deflection.

Table 6 – Averaged Effective Moment of Inertia Calculations

For Frame Strip

Span	zone	I_g, mm^4 ($\times 10^9$)	I_{cr}, mm^4 ($\times 10^9$)	M_a, kN.m			M_{cr}, kN.m	I_e, mm^4 ($\times 10^9$)			$I_{e,avg}$, mm^4 ($\times 10^9$)		
				D	D + LL_{Sus}	D + L_{full}		D	D + LL_{Sus}	D + L_{full}	D	D + LL_{Sus}	D + L_{full}
Ext	Left	3.65	3.15	-44.85	-44.85	-96.52	21.88	3.21	3.21	3.16	8.23	8.23	2.50
	Midspan	9.95	1.63	39.07	39.07	84.08	37.73	9.13	9.13	2.39			
	Right	3.65	3.15	-83.60	-83.60	-179.92	21.88	3.16	3.16	3.15			
Int	Left	3.65	3.15	-75.96	-75.96	-163.49	21.88	3.16	3.16	3.15	7.92	7.92	3.79
	Mid	9.95	1.63	26.43	26.43	63.56	37.73	9.95	9.95	4.06			
	Right	3.65	3.15	-75.96	-75.96	-163.49	21.88	3.16	3.16	3.15			

Deflections in two-way slab systems shall be calculated taking into account size and shape of the panel, conditions of support, and nature of restraints at the panel edges. For immediate deflections two-way slab systems the midpanel deflection is computed as the sum of deflection at midspan of the column strip or column line in one direction (Δ_{cx} or Δ_{cy}) and deflection at midspan of the middle strip in the orthogonal direction (Δ_{mx} or Δ_{my}). Figure 21 shows the deflection computation for a rectangular panel. The average Δ for panels that have different properties in the two direction is calculated as follows:

$$\Delta = \frac{(\Delta_{cx} + \Delta_{my}) + (\Delta_{cy} + \Delta_{mx})}{2}$$ ***<u>PCA Notes on ACI 318-11 (9.5.3.4 Eq. 8)</u>***

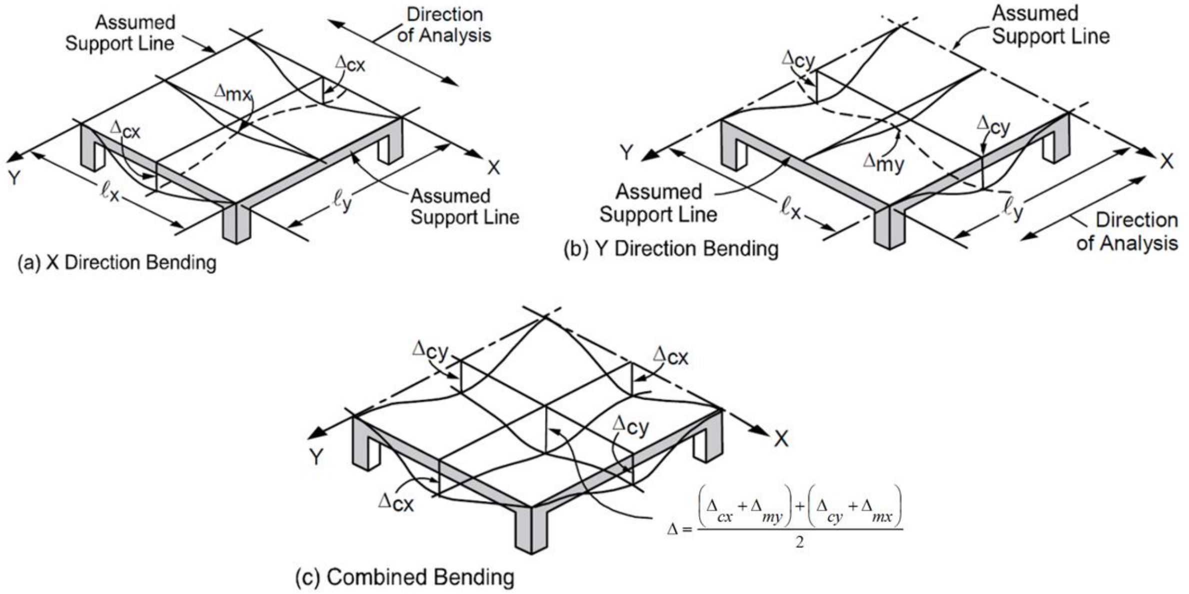


Figure 21 – Deflection Computation for a rectangular Panel

To calculate each term of the previous equation, the following procedure should be used. Figure 22 shows the procedure of calculating the term Δ_{cx}. same procedure can be used to find the other terms.

$$\Delta_{cx} = \Delta_{cx,fixed} + \Delta\theta_{cx,R} + \Delta\theta_{cx,L}$$

$$\Delta_{c,fixed} = LDF_c \times \Delta_{frame,fixed} \times \left(\frac{I_{frame}}{I_c}\right)_g$$

$$LDF_c = \frac{LDF^+ + \frac{LDF_l^- + LDF_R^-}{2}}{2}$$

$$\Delta_{frame,fixed} = \frac{wl^4}{384E_cI_{frame,averaged}}$$

$$\Delta\theta_{c,R} = \theta_{c,R}\left(\frac{l}{8}\right)\left(\frac{I_g}{I_e}\right)_{column}$$

$$\theta_{c,R} = \frac{(M_{net,R})_{column}}{K_{ec}}$$

$$\Delta\theta_{c,L} = \theta_{c,L}\left(\frac{l}{8}\right)\left(\frac{I_g}{I_e}\right)_{column}$$

$$\theta_{c,L} = \frac{(M_{net,L})_{column}}{K_{ec}}$$

Figure 22 –Δ_{cx} calculation procedure

For exterior span - service dead load case:

$$\Delta_{frame,fixed} = \frac{wl^4}{384E_cI_{frame,averaged}}$$

PCA Notes on ACI 318-11 (9.5.3.4 Eq. 10)

Where:

$\Delta_{frame, fixed}$ = Deflection of column strip assuing fixed end condition.

$$w = \text{slab weight + beam weight} = \left(\frac{24\times155}{1000}+\frac{24\times(500-155)\times350}{6.5\times1000}\right)(6.5) = 27.08\text{ kN/m}$$

$$E_{cs} = (3,300\sqrt{f_c^{'}}+6,900)\left(\frac{\gamma_c}{2,300}\right)^{1.5} = (3,300\sqrt{25}+6,900)\left(\frac{2,447}{2,300}\right)^{1.5} = 25,684\text{ MPa}$$

CSA A23.3-14(8.6.2.2)

$I_{frame,averaged}$ = The averaged effective moment of inertia ($I_{e,avg}$) for the frame strip for service dead load case from Table 6 = 8.23 x 10^9 mm^4

$$\Delta_{frame, fixed} = \frac{(27.08)(5500-450)^4}{384(25,684)(8.23\times10^9)} = 0.217\text{ mm}$$

$$\Delta_{c, fixed} = LDF_c \times \Delta_{frame, fixed} \times \left(\frac{I_{frame}}{I_c}\right)_g$$

PCA Notes on ACI 318-11 (9.5.3.4 Eq. 11)

Where LDF_c is the load distribution factor for the column strip. The load distribution factor for the column strip can be found from the following equation:

$$LDF_c = \frac{LDF^+ + \dfrac{LDF_l^- + LDF_R^-}{2}}{2}$$

And the load distribution factor for the middle strip can be found from the following equation:

$$LDF_m = 1 - LDF_c$$

For the end span, LDF for exterior negative region (LDF_L^-), interior negative region (LDF_R^-), and positive region (LDF_L^+) are 1.00, 0.727, and 0.727, respectively (From Table 2 of this document). Thus, the load distribution factor for the column strip for the end span is given by:

$$LDF_c = \frac{0.727 + \dfrac{1.00+0.727}{2}}{2} = 0.795$$

$I_{c,g}$ = The gross moment of inertia (I_g) for the column strip (for T section) = 7.93 x 10^9 mm^4

$I_{frame,g}$ = The gross moment of inertia (I_g) for the frame strip (for T section) = 9.95 x 10^9 mm^4

$$\Delta_{c, fixed} = 0.795\times0.217\times\frac{9.95\times10^9}{7.93\times10^9} = 0.217\text{ mm}$$

$$\theta_{c,L} = \frac{(M_{net,L})_{frame}}{K_{ec}}$$

PCA Notes on ACI 318-11 (9.5.3.4 Eq. 12)

Where:

$\theta_{c,L}$ = Rotation of the span left support.

$(M_{net,L})_{frame} = 4.49 \times 10^7$ N.mm = Net frame strip negative moment of the left support.

K_{ec} = effective column stiffness for exterior column.

= 3.05 x 10^{11} N.mm/rad (calculated previously).

$$\theta_{c,L} = \frac{4.49 \times 10^7}{3.05 \times 10^{11}} = 0.00015 \text{ rad}$$

$$\Delta\theta_{c,L} = \theta_{c,L}\left(\frac{l}{8}\right)\left(\frac{I_g}{I_e}\right)_{frame}$$

PCA Notes on ACI 318-11 (9.5.3.4 Eq. 14)

Where:

$\Delta\theta_{c,L}$ = Midspan deflection due to rotation of left support.

$\left(\frac{I_g}{I_e}\right)_{frame}$ = Gross-to-effective moment of inertia ratio for frame strip.

$$\Delta\theta_{c,L} = 0.00015 \times \frac{5500-450}{8} \times \frac{9.95 \times 10^9}{8.23 \times 10^9} = 0.112 \text{ mm}$$

$$\theta_{c,R} = \frac{(M_{net,R})_{frame}}{K_{ec}} = \frac{(8.36-7.60) \times 10^7}{2.45 \times 10^{11}} = 0.00003 \text{ rad}$$

Where

$\theta_{c,R}$ = Rotation of the end span right support.

$(M_{net,R})_{frame}$ = Net frame strip negative moment of the right support.

K_{ec} = effective column stiffness for interior column.

= 2.45 x 10^{11} N.mm/rad (calculated previously).

$$\Delta\theta_{c,R} = \theta_{c,R}\left(\frac{l}{8}\right)\left(\frac{I_g}{I_e}\right)_{frame} = 0.00003 \times \frac{5500-450}{8} \times \frac{9.95 \times 10^9}{8.23 \times 10^9} = 0.024 \text{ mm}$$

Where:

$\Delta\theta_{c,R}$ = Midspan delfection due to rotation of right support.

$$\Delta_{cx} = \Delta_{cx,\,fixed} + \Delta\theta_{cx,R} + \Delta\theta_{cx,L}$$

PCA Notes on ACI 318-11 (9.5.3.4 Eq. 9)

$$\Delta_{cx} = 0.217 + 0.112 + 0.024 = 0.353 \text{ mm}$$

Following the same procedure, Δ_{mx} can be calculated for the middle strip. This procedure is repeated for the equivalent frame in the orthogonal direction to obtain Δ_{cy}, and Δ_{my} for the end and middle spans for the other load levels ($D+LL_{sus}$ *and* $D+LL_{full}$).

Assuming square panel, $\Delta_{cx} = \Delta_{cy} = 0.009$ in. and $\Delta_{mx} = \Delta_{my} = 0.021$ in.

The average Δ for the corner panel is calculated as follows:

$$\Delta = \frac{\left(\Delta_{cx} + \Delta_{my}\right) + \left(\Delta_{cy} + \Delta_{mx}\right)}{2} = \left(\Delta_{cx} + \Delta_{my}\right) = \left(\Delta_{cy} + \Delta_{mx}\right) = 0.009 + 0.021 = 0.030 \text{ in.}$$

Table 7 - Instantaneous Deflections

Column Strip

Span	LDF	D: $\Delta_{frame-fixed}$, mm	$\Delta_{c-fixed}$, mm	θ_{c1}, rad	θ_{c2}, rad	$\Delta\theta_{c1}$, mm	$\Delta\theta_{c2}$, mm	Δ_{cx}, mm
Ext	0.795	0.217	0.217	0.00015	0.00003	0.112	0.024	0.353
Int	0.727	0.225	0.206	0.00003	0.00003	0.025	0.025	0.156

Span	LDF	D+LL$_{sus}$: $\Delta_{frame-fixed}$, mm	$\Delta_{c-fixed}$, mm	θ_{c1}, rad	θ_{c2}, rad	$\Delta\theta_{c1}$, mm	$\Delta\theta_{c2}$, mm	Δ_{cx}, mm
Ext	0.795	0.217	0.217	0.00015	0.00003	0.112	0.024	0.353
Int	0.727	0.225	0.206	0.00003	0.00003	0.025	0.025	0.156

Span	LDF	D+LL$_{full}$: $\Delta_{frame-fixed}$, mm	$\Delta_{c-fixed}$, mm	θ_{c1}, rad	θ_{c2}, rad	$\Delta\theta_{c1}$, mm	$\Delta\theta_{c2}$, mm	Δ_{cx}, mm
Ext	0.795	1.537	1.534	0.00032	0.00007	0.795	0.168	2.497
Int	0.727	1.014	0.925	0.00007	0.00007	0.111	0.111	0.703

Span	LDF	LL: Δ_{cx}, mm
Ext	0.795	2.144
Int	0.727	0.547

Middle Strip

LDF	D: $\Delta_{frame-fixed}$, mm	$\Delta_{m-fixed}$, mm	θ_{m1}, rad	θ_{m2}, rad	$\Delta\theta_{m1}$, mm	$\Delta\theta_{m2}$, mm	Δ_{mx}, mm
0.205	0.217	0.381	0.00015	0.00003	0.112	0.024	0.517
0.273	0.225	0.528	0.00003	0.00003	0.025	0.025	0.479

LDF	D+LL$_{sus}$: $\Delta_{frame-fixed}$, mm	$\Delta_{m-fixed}$, mm	θ_{m1}, rad	θ_{m2}, rad	$\Delta\theta_{m1}$, mm	$\Delta\theta_{m2}$, mm	Δ_{mx}, mm
0.205	0.217	0.381	0.00015	0.00003	0.112	0.024	0.517
0.273	0.225	0.528	0.00003	0.00003	0.025	0.025	0.479

LDF	D+LL$_{full}$: $\Delta_{frame-fixed}$, mm	$\Delta_{m-fixed}$, mm	θ_{m1}, rad	θ_{m2}, rad	$\Delta\theta_{m1}$, mm	$\Delta\theta_{m2}$, mm	Δ_{mx}, mm
0.205	1.537	2.700	0.00032	0.00007	0.795	0.168	3.663
0.273	1.014	2.375	0.00007	0.00007	0.111	0.111	2.153

LDF	LL: Δ_{mx}, mm
0.205	3.146
0.273	1.674

5.2. Time-Dependent (Long-Term) Deflections (Δ_{lt})

The additional time-dependent (long-term) deflection resulting from creep and shrinkage (Δ_{cs}) may be estimated as follows:

$$\Delta_{cs} = \lambda_{\Delta} \times (\Delta_{sust})_{Inst}$$ ***PCA Notes on ACI 318-11 (9.5.2.5 Eq. 4)***

The total time-dependent (long-term) deflection is calculated as:

$$(\Delta_{total})_{lt} = (\Delta_{sust})_{Inst} \times (1+\lambda_{\Delta}) + [(\Delta_{total})_{Inst} - (\Delta_{sust})_{Inst}]$$ ***CSA A23.3-04 (N9.8.2.5)***

Where:

$(\Delta_{sust})_{Inst} =$ Immediate (instantaneous) deflection due to sustained load, in.

$$\lambda_{\Delta} = \frac{\xi}{1+50\rho'}$$ ***ACI 318-14 (24.2.4.1.1)***

$(\Delta_{total})_{lt} =$ Time-dependent (long-term) total delfection, in.

$(\Delta_{total})_{Inst} =$ Total immediate (instantaneous) deflection, in.

For the exterior span

ξ = 2, consider the sustained load duration to be 60 months or more. ***ACI 318-14 (Table 24.2.4.1.3)***

ρ' = 0, conservatively.

$$\lambda_{\Delta} = \frac{2}{1+50\times 0} = 2$$

$$\Delta_{cs} = 2 \times 0.353 = 0.706 \text{ mm}$$

$$(\Delta_{total})_{lt} = 0.353 \times (1+2) + (2.497 - 0.353) = 3.203 \text{ mm}$$

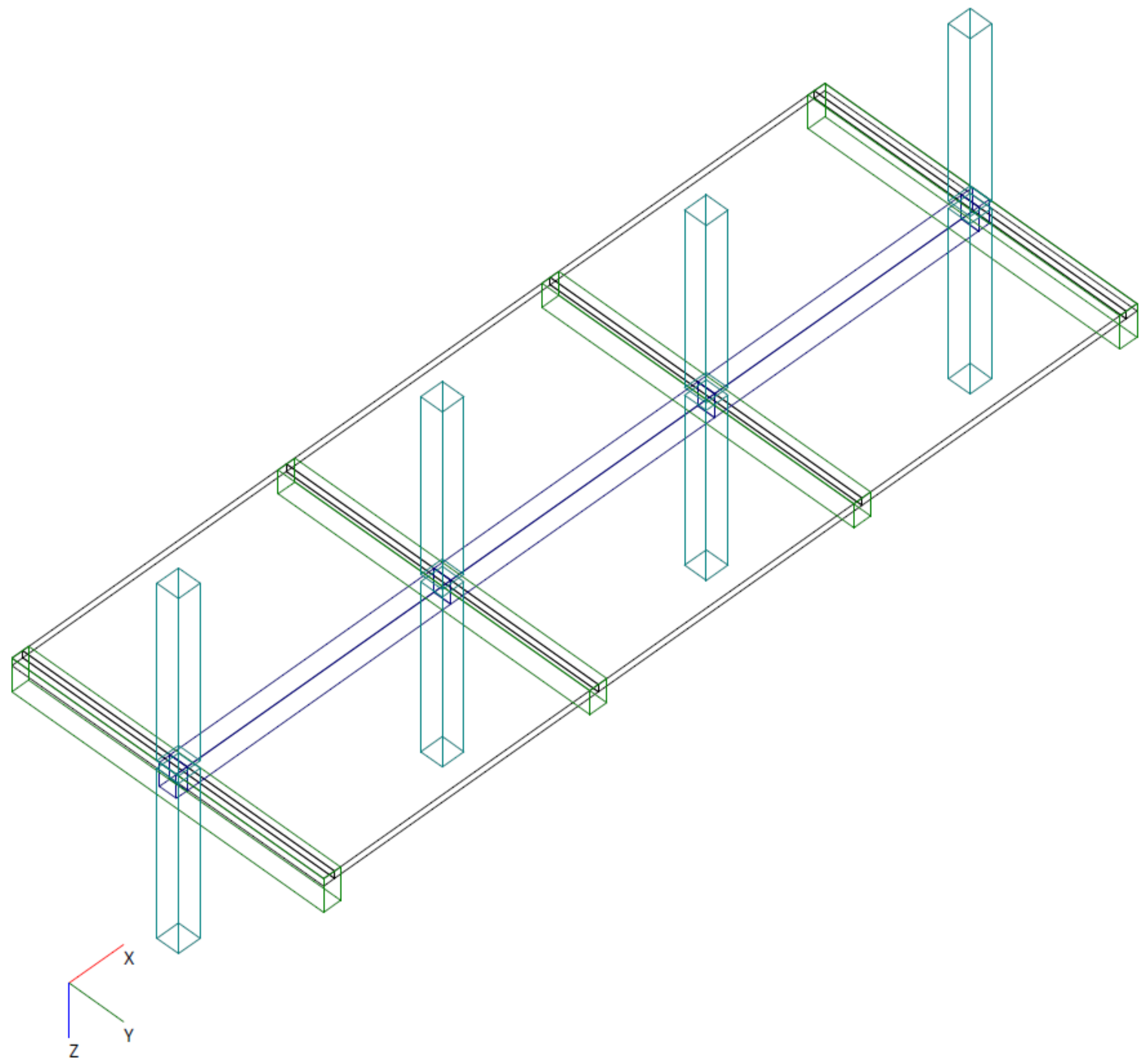
Table 8 shows long-term deflections for the exterior and interior spans for the analysis in the x-direction, for column and middle strips.

Table 8 - Long-Term Deflections					
Column Strip					
Span	**$(\Delta_{sust})_{Inst}$, mm**	**λ_Δ**	**Δ_{cs}, mm**	**$(\Delta_{total})_{Inst}$, mm**	**$(\Delta_{total})_{lt}$, mm**
Exterior	0.353	2.000	0.706	2.497	3.203
Interior	0.156	2.000	0.312	0.703	1.015
Middle Strip					
Exterior	0.517	2.000	1.034	3.663	4.697
Interior	0.479	2.000	0.958	2.153	3.111

6. spSlab Software Program Model Solution

spSlab program utilizes the Elastic Frame Method described and illustrated in details here for modeling, analysis and design of two-way concrete floor slab systems. spSlab uses the exact geometry and boundary conditions provided as input to perform an elastic stiffness (matrix) analysis of the equivalent frame taking into account the torsional stiffness of the slabs framing into the column. It also takes into account the complications introduced by a large number of parameters such as vertical and torsional stiffness of transverse beams, the stiffening effect of drop panels, column capitals, and effective contribution of columns above and below the floor slab using the of equivalent column concept ***(CSA A23.3-14 (13.8.2.6))***.

spSlab Program models the elastic frame as a design strip. The design strip is, then, separated by spSlab into column and middle strips. The program calculates the internal forces (Shear Force & Bending Moment), moment and shear capacity vs. demand diagrams for column and middle strips, instantaneous and long-term deflection results, and required flexural reinforcement for column and middle strips. The graphical and text results will be provided from the spSlab model in a future revision to this document.



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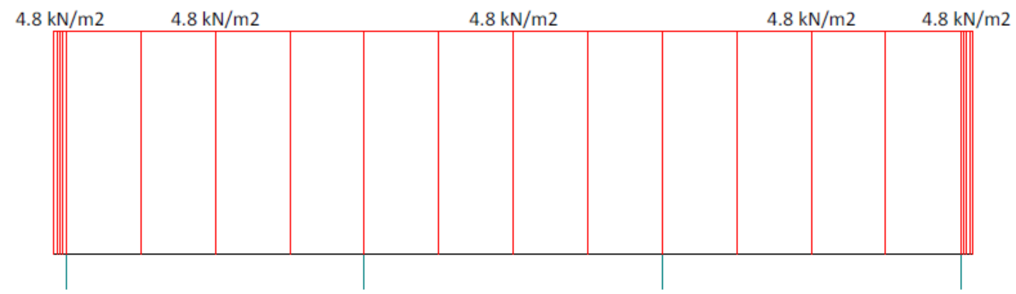
Frame: Interior Frame

Engineer: SP

Code: CSA A23.3-14

Date: 10/03/18

Time: 09:11:06



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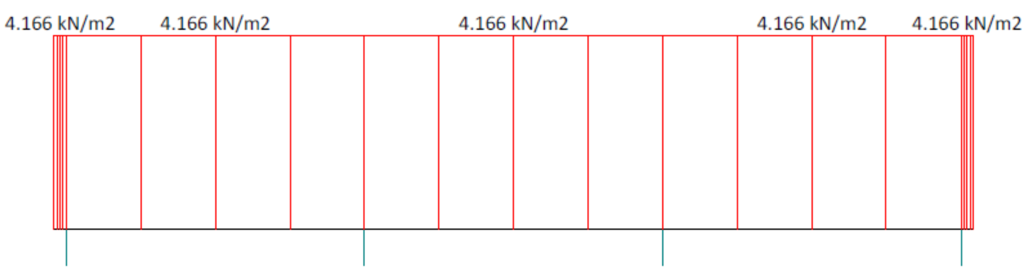

CASE: Dead

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Project: Two-Way Slab With Beams Spanning Between Supports

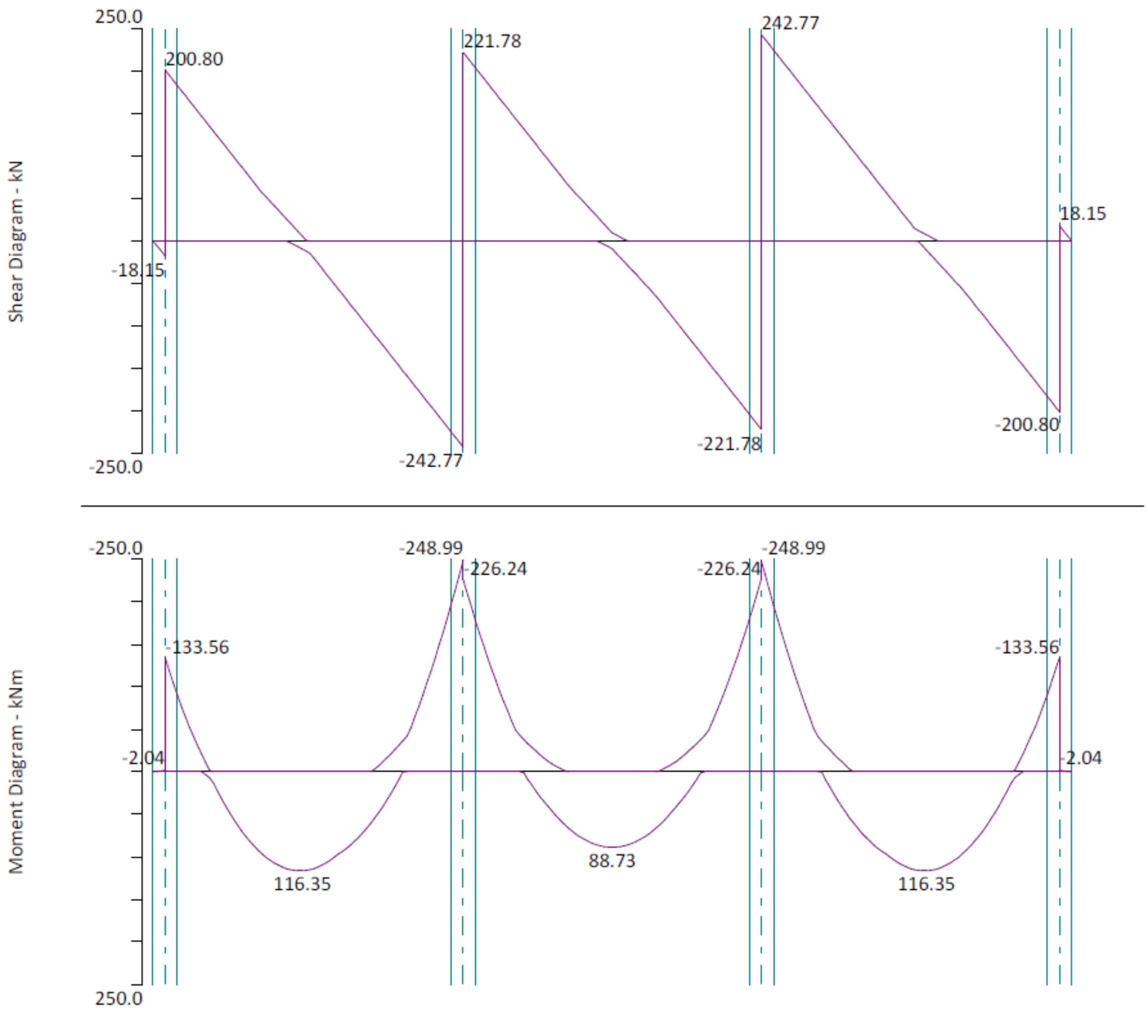
Frame: Interior Frame

Engineer: SP

Code: CSA A23.3-14

Date: 10/03/18

Time: 09:13:13



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Frame: Interior Frame

Engineer: SP

Code: CSA A23.3-14

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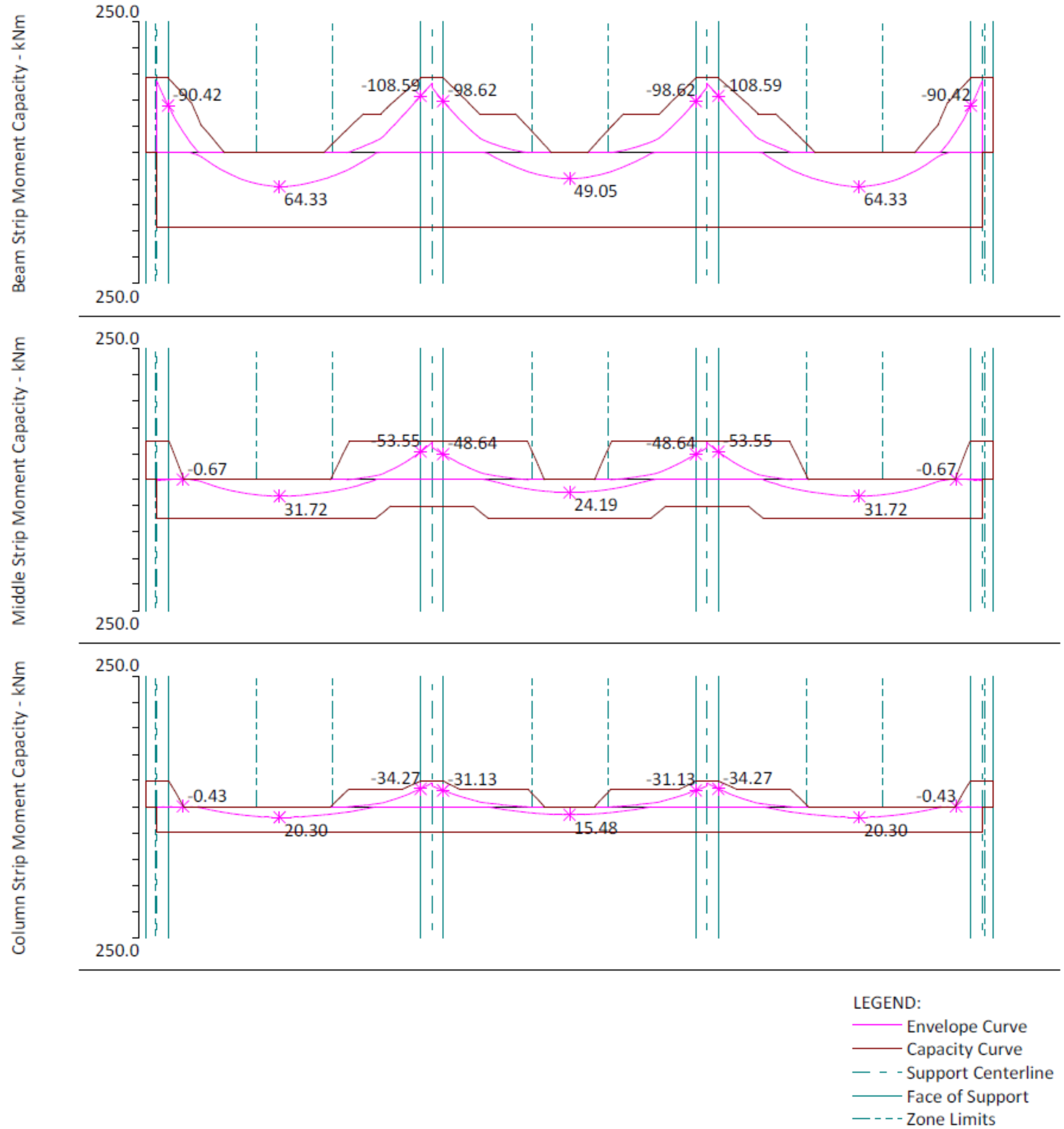

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Project: Two-Way Slab With Beams Spanning Between Supports

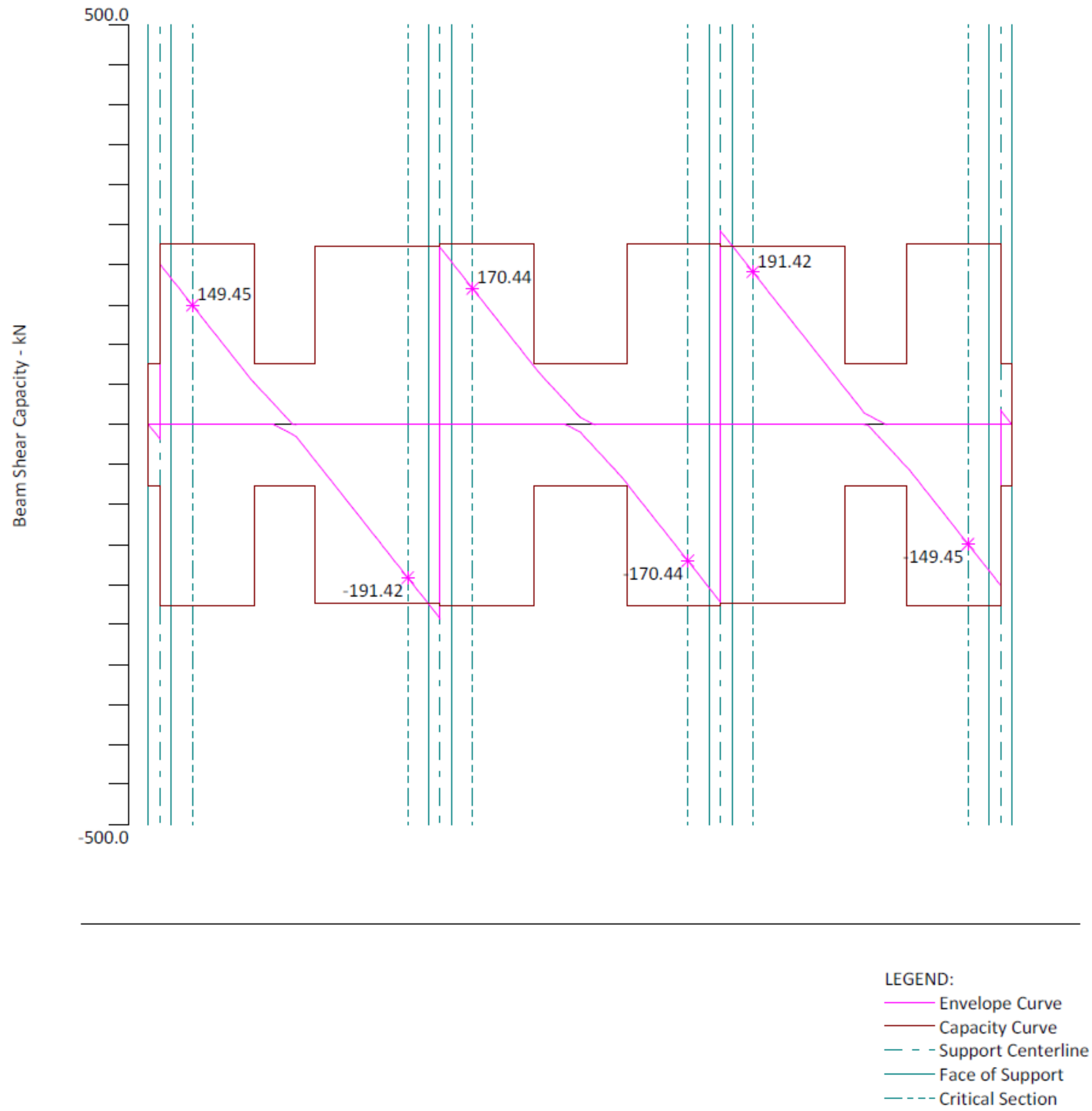
Frame: Interior Frame

Engineer: SP

Code: CSA A23.3-14

Date: 10/03/18

Time: 09:17:28



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Project: Two-Way Slab With Beams Spanning Between Supports

Frame: Interior Frame

Engineer: SP

Code: CSA A23.3-14

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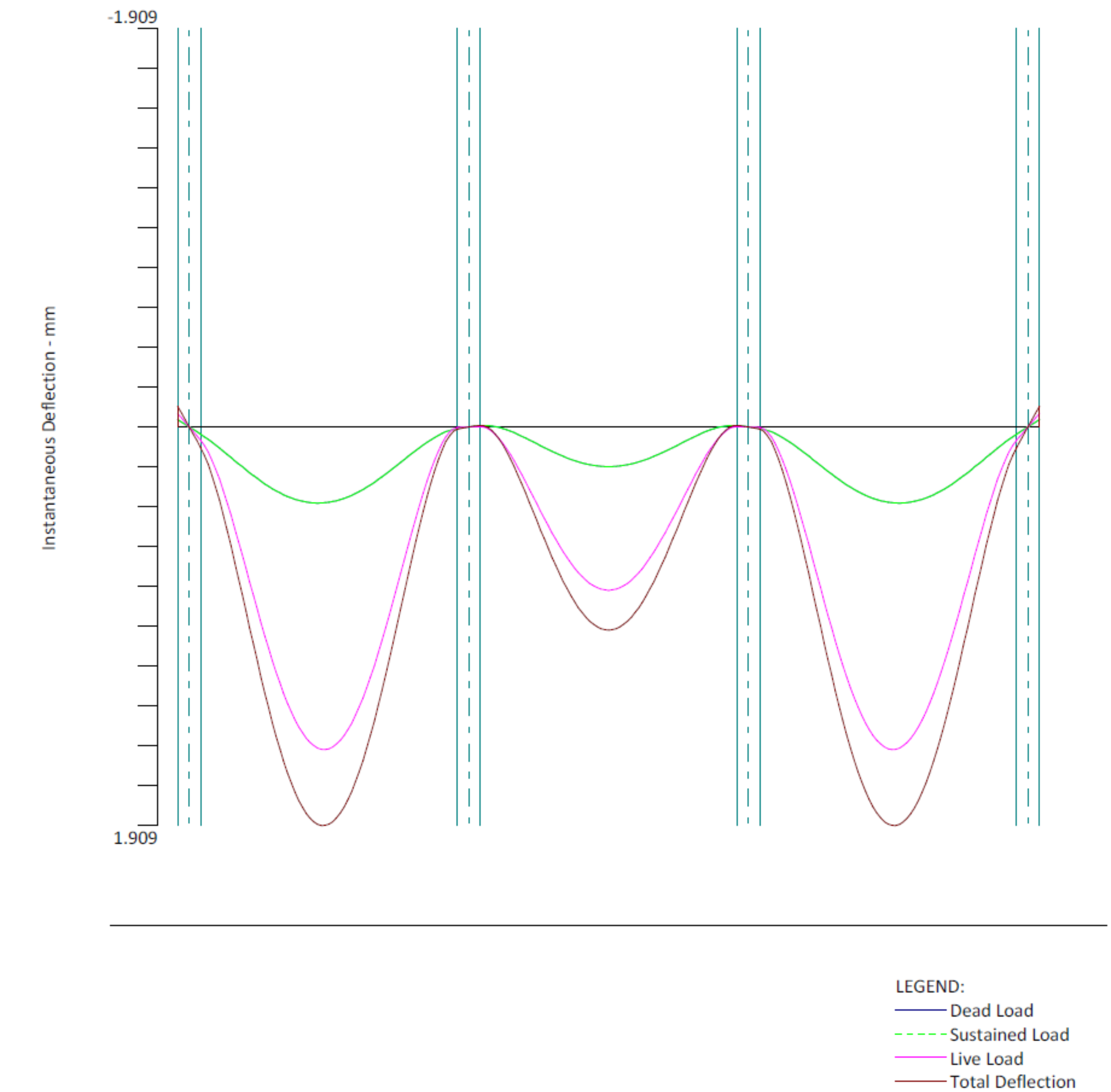

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Frame: Interior Frame

Engineer: SP

Code: CSA A23.3-14

Date: 10/03/18

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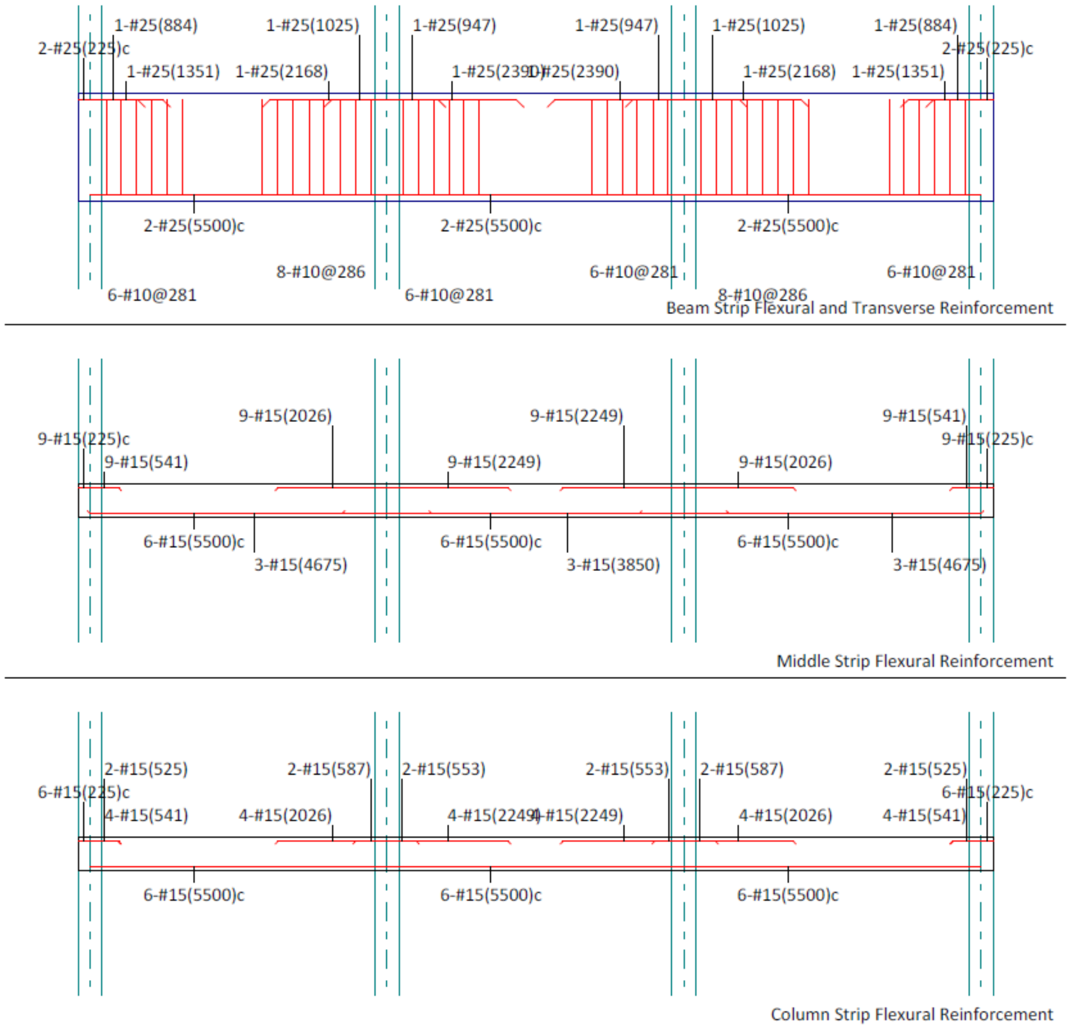

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Frame: Interior Frame

Engineer: SP

Code: CSA A23.3-14

Date: 10/03/18

Time: 09:35:57

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         ================================================================================
                                       spSlab v5.00 (TM)
                    A Computer Program for Analysis, Design, and Investigation of
                     Reinforced Concrete Beams, One-way and Two-way Slab Systems
                             Copyright © 2003-2015, STRUCTUREPOINT, LLC
                                        All rights reserved
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General Information
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   Frame: Interior Frame
   Engineer: SP
   Code: CSA A23.3-14
   Reinforcement Database: CSA G30.18
   Mode: Design
   Number of supports = 4 + Left cantilever + Right cantilever
   Floor System: Two-Way

   Live load pattern ratio = 75%
   Minimum free edge distance for punching shear = 5 times slab effective depth.
   Circular critical section around circular supports used (if possible).
   Deflections are based on cracked section properties.
   In negative moment regions, Ig and Mcr DO NOT include flange/slab contribution (if available)
   Long-term deflections are calculated for load duration of 60 months.
   0% of live load is sustained.
   Compression reinforcement calculations NOT selected.
   Default incremental rebar design selected.
   Combined M-V-T reinforcement design NOT selected.
   User-defined slab strip widths NOT selected.
   User-defined distribution factors NOT selected.
   One-way shear in drop panel NOT selected.
   Distribution of shear to strips NOT selected.
   Beam T-section design NOT selected.
   Longitudinal beam contribution in negative reinforcement design over support NOT selected.
   Transverse beam contribution in negative reinforcement design over support NOT selected.

Material Properties
===================
              Slabs|Beams       Columns
             ------------   ------------
   wc    =        2447.3         2447.3 kg/m3
   f'c   =            25             25 MPa
   Ec    =         25684          25684 MPa
   fr    =           1.5              3 MPa
   Precast concrete construction is not selected.

   fy    =           400 MPa, Bars are not epoxy-coated
   fyt   =           400 MPa
   Es    =        200000 MPa

Reinforcement Database
======================
   Units: Db (mm), Ab (mm^2), Wb (kg/m)
   Size      Db       Ab       Wb    Size      Db       Ab       Wb
   ---- -------- -------- --------   ---- -------- -------- --------
    #10      11      100        1     #15      16      200        2
    #20      20      300        2     #25      25      500        4
```

#30	30	700	5	#35	36	1000	8
#45	44	1500	12	#55	56	2500	20

Span Data

Slabs

Units: L1, wL, wR, L2L, L2R (m); t, Hmin (mm)

Span	Loc	L1	t	wL	wR	L2L	L2R	Hmin	
1	Int	0.225	155	3.250	3.250	6.500	6.500	---	LC *i
2	Int	5.500	155	3.250	3.250	6.500	6.500	153	
3	Int	5.500	155	3.250	3.250	6.500	6.500	153	
4	Int	5.500	155	3.250	3.250	6.500	6.500	153	
5	Int	0.225	155	3.250	3.250	6.500	6.500	---	RC *i

NOTES:
Deflection check required for panels where code-specified Hmin for two-way construction doesn't apply due to:
*i - cantilever end span (LC, RC) support condition

Ribs and Longitudinal Beams

Units: b, h, Sp (mm)

	Ribs			Beams		
Span	b	h	Sp	b	h	Offset
1	0	0	0	350	500	0
2	0	0	0	350	500	0
3	0	0	0	350	500	0
4	0	0	0	350	500	0
5	0	0	0	350	500	0

Support Data

Columns

Units: c1a, c2a, c1b, c2b (mm); Ha, Hb (m)

Supp	c1a	c2a	Ha	c1b	c2b	Hb	Red%
1	450	450	3.700	450	450	3.700	100
2	450	450	3.700	450	450	3.700	100
3	450	450	3.700	450	450	3.700	100
4	450	450	3.700	450	450	3.700	100

Transverse Beams

Units: b, h, Ecc (mm)

Supp	b	h	Ecc
1	350	700	-50
2	350	500	0
3	350	500	0
4	350	700	50

Boundary Conditions

Units: Kz (kN/mm); Kry (kN-mm/rad)

Supp	Spring Kz	Spring Kry	Far End A	Far End B
1	0	0	Fixed	Fixed
2	0	0	Fixed	Fixed
3	0	0	Fixed	Fixed
4	0	0	Fixed	Fixed

Load Data

Load Cases and Combinations

Case	Dead	Live
Type	DEAD	LIVE
U1	1.250	1.500

Area Loads

Units: Wa (kN/m2)

Case/Patt	Span	Wa
Dead	1	4.17
	2	4.17
	3	4.17
	4	4.17
	5	4.17
Live	1	4.80
	2	4.80
	3	4.80
	4	4.80
	5	4.80

```
   Live/Odd      1          3.60
                 3          3.60
                 5          3.60
   Live/Even     2          3.60
                 4          3.60
   Live/S1       1          3.60
                 2          3.60
   Live/S2       2          3.60
                 3          3.60
   Live/S3       3          3.60
                 4          3.60
   Live/S4       4          3.60
                 5          3.60

Reinforcement Criteria
======================
   Slabs and Ribs
   --------------
                 _____Top bars___  __Bottom bars___
                    Min      Max      Min      Max
                 -------  -------  -------  -------
     Bar Size       #15      #35      #15      #35
     Bar spacing     25      457       25      457 mm
     Reinf ratio   0.14     5.00     0.14     5.00 %
     Cover           20                20          mm
     There is NOT more than 300 mm of concrete below top bars.

   Beams
   -----
                 _____Top bars___  __Bottom bars___  ____Stirrups____
                    Min      Max      Min      Max      Min      Max
                 -------  -------  -------  -------  -------  -------
     Bar Size       #25      #30      #25      #30      #10      #10
     Bar spacing     25      457       25      457      152      457 mm
     Reinf ratio   0.14     5.00     0.14     5.00 %
     Cover           30                30          mm
     Layer dist.     25                25          mm
     No. of legs                                          2        6
     Side cover                                          38          mm
     1st Stirrup                                         76          mm
     There is NOT more than 300 mm of concrete below top bars.
```

```
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spSlab v5.00 (TM)
A Computer Program for Analysis, Design, and Investigation of
Reinforced Concrete Beams, One-way and Two-way Slab Systems

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[2] DESIGN RESULTS*

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*Unless otherwise noted, all results are in the direction of analysis only. Another analysis in the perpendicular direction has to be carried out for two-way slab systems.

Strip Widths and Distribution Factors

Units: Width (m).

Span	Strip	Width Left**	Width Right**	Width Bottom*	Moment Factor Left**	Moment Factor Right**	Moment Factor Bottom*
1	Column	2.40	2.40	2.40	0.000	0.000	0.185
	Middle	3.75	3.75	3.75	0.000	0.000	0.207
	Beam	0.35	0.35	0.35	1.000	1.000	0.608
2	Column	2.40	2.40	2.40	0.000	0.174	0.174
	Middle	3.75	3.75	3.75	0.000	0.273	0.273
	Beam	0.35	0.35	0.35	1.000	0.553	0.553
3	Column	2.40	2.40	2.40	0.174	0.174	0.174
	Middle	3.75	3.75	3.75	0.273	0.273	0.273
	Beam	0.35	0.35	0.35	0.553	0.553	0.553
4	Column	2.40	2.40	2.40	0.174	0.000	0.174
	Middle	3.75	3.75	3.75	0.273	0.000	0.273
	Beam	0.35	0.35	0.35	0.553	1.000	0.553
5	Column	2.40	2.40	2.40	0.000	0.000	0.185
	Middle	3.75	3.75	3.75	0.000	0.000	0.207
	Beam	0.35	0.35	0.35	1.000	1.000	0.608

*Used for bottom reinforcement. **Used for top reinforcement.

Top Reinforcement

Units: Width (m), Mmax (kNm), Xmax (m), As (mm^2), Sp (mm)

Span	Strip	Zone	Width	Mmax	Xmax	AsMin	AsMax	AsReq	SpProv	Bars
1	Column	Left	2.40	0.00	0.000	744	6835	0	400	6-#15 *3 *5
		Midspan	2.40	0.00	0.093	744	6835	0	400	6-#15 *3 *5
		Right	2.40	0.00	0.186	744	6835	0	400	6-#15 *3 *5
	Middle	Left	3.75	0.00	0.000	1163	10680	0	417	9-#15 *3 *5
		Midspan	3.75	0.00	0.093	1163	10680	0	417	9-#15 *3 *5
		Right	3.75	0.00	0.186	1163	10680	0	417	9-#15 *3 *5
	Beam	Left	0.35	0.20	0.065	438	3590	1	220	2-#25 *3
		Midspan	0.35	0.64	0.121	438	3590	4	220	2-#25 *3
		Right	0.35	1.45	0.186	438	3590	9	220	2-#25 *3
2	Column	Left	2.40	0.43	0.522	744	6835	10	400	6-#15 *3 *5
		Midspan	2.40	0.00	2.750	0	6835	0	0	---
		Right	2.40	34.27	5.275	744	6835	822	400	6-#15 *5

	Middle	Left	3.75	0.67	0.522	1163	10680	16	417	9-#15 *3 *5
		Midspan	3.75	0.00	2.750	0	10680	0	0	---
		Right	3.75	53.55	5.275	1163	10680	1285	417	9-#15 *5
	Beam	Left	0.35	90.42	0.225	438	3590	612	220	2-#25
		Midspan	0.35	0.00	2.750	0	3590	0	0	---
		Right	0.35	108.59	5.275	438	3590	743	220	2-#25
3	Column	Left	2.40	31.13	0.225	744	6835	744	400	6-#15 *5
		Midspan	2.40	0.00	2.750	0	6835	0	0	---
		Right	2.40	31.13	5.275	744	6835	744	400	6-#15 *5
	Middle	Left	3.75	48.64	0.225	1163	10680	1163	417	9-#15 *5
		Midspan	3.75	0.00	2.750	0	10680	0	0	---
		Right	3.75	48.64	5.275	1163	10680	1163	417	9-#15 *5
	Beam	Left	0.35	98.62	0.225	438	3590	670	220	2-#25
		Midspan	0.35	0.00	2.750	0	3590	0	0	---
		Right	0.35	98.62	5.275	438	3590	670	220	2-#25
4	Column	Left	2.40	34.27	0.225	744	6835	822	400	6-#15 *5
		Midspan	2.40	0.00	2.750	0	6835	0	0	---
		Right	2.40	0.43	4.978	744	6835	10	400	6-#15 *3 *5
	Middle	Left	3.75	53.55	0.225	1163	10680	1285	417	9-#15 *5
		Midspan	3.75	0.00	2.750	0	10680	0	0	---
		Right	3.75	0.67	4.978	1163	10680	16	417	9-#15 *3 *5
	Beam	Left	0.35	108.59	0.225	438	3590	743	220	2-#25
		Midspan	0.35	0.00	2.750	0	3590	0	0	---
		Right	0.35	90.42	5.275	438	3590	612	220	2-#25
5	Column	Left	2.40	0.00	0.039	744	6835	0	400	6-#15 *3 *5
		Midspan	2.40	0.00	0.132	744	6835	0	400	6-#15 *3 *5
		Right	2.40	0.00	0.225	744	6835	0	400	6-#15 *3 *5
	Middle	Left	3.75	0.00	0.039	1163	10680	0	417	9-#15 *3 *5
		Midspan	3.75	0.00	0.132	1163	10680	0	417	9-#15 *3 *5
		Right	3.75	0.00	0.225	1163	10680	0	417	9-#15 *3 *5
	Beam	Left	0.35	1.45	0.039	438	3590	9	220	2-#25 *3
		Midspan	0.35	0.64	0.104	438	3590	4	220	2-#25 *3
		Right	0.35	0.20	0.160	438	3590	1	220	2-#25 *3

NOTES:
*3 - Design governed by minimum reinforcement.
*5 - Number of bars governed by maximum allowable spacing.

Top Bar Details
==============

Units: Length (m)

		Left				Continuous		Right			
Span	Strip	Bars	Length	Bars	Length	Bars	Length	Bars	Length	Bars	Length
1	Column	---		---		6-#15	0.23	---		---	
	Middle	---		---		9-#15	0.23	---		---	
	Beam	---		---		2-#25	0.23	---		---	
2	Column	4-#15*	0.54	2-#15*	0.53	---		4-#15	2.03	2-#15*	0.59
	Middle	9-#15*	0.54	---		---		9-#15	2.03	---	
	Beam	1-#25	1.35	1-#25	0.88	---		1-#25	2.17	1-#25*	1.02
3	Column	4-#15	2.25	2-#15*	0.55	---		4-#15	2.25	2-#15*	0.55
	Middle	9-#15	2.25	---		---		9-#15	2.25	---	
	Beam	1-#25	2.39	1-#25*	0.95	---		1-#25	2.39	1-#25*	0.95
4	Column	4-#15	2.03	2-#15*	0.59	---		4-#15*	0.54	2-#15*	0.53
	Middle	9-#15	2.03	---		---		9-#15*	0.54	---	
	Beam	1-#25	2.17	1-#25*	1.02	---		1-#25	1.35	1-#25	0.88
5	Column	---		---		6-#15	0.23	---		---	
	Middle	---		---		9-#15	0.23	---		---	
	Beam	---		---		2-#25	0.23	---		---	

NOTES:
* - Bar cut-off location shall be manually checked for compliance with CSA A23.3, 11.2.13.

Top Bar Development Lengths
==========================

Units: Length (mm)

		Left				Continuous		Right			
Span	Strip	Bars	DevLen	Bars	DevLen	Bars	DevLen	Bars	DevLen	Bars	DevLen
1	Column	---		---		6-#15	300.00	---		---	
	Middle	---		---		9-#15	300.00	---		---	
	Beam	---		---		2-#25	300.00	---		---	
2	Column	4-#15	300.00	2-#15	300.00	---		4-#15	362.13	2-#15	362.13
	Middle	9-#15	300.00	---		---		9-#15	377.22	---	

	Beam	1-#25	658.69	1-#25	658.69	---		1-#25	799.85	1-#25	799.85
3	Column	4-#15	327.78	2-#15	327.78	---		4-#15	327.78	2-#15	327.78
	Middle	9-#15	341.44	---		---		9-#15	341.44	---	
	Beam	1-#25	722.00	1-#25	722.00	---		1-#25	722.00	1-#25	722.00
4	Column	4-#15	362.13	2-#15	362.13	---		4-#15	300.00	2-#15	300.00
	Middle	9-#15	377.22	---		---		9-#15	300.00	---	
	Beam	1-#25	799.85	1-#25	799.85	---		1-#25	658.69	1-#25	658.69
5	Column	---		---		6-#15	300.00	---		---	
	Middle	---		---		9-#15	300.00	---		---	
	Beam	---		---		2-#25	300.00	---		---	

Band Reinforcement at Supports

Units: Width (mm), As (mm^2)

Supp	Width<C>	Width<B>	Width<S>	As<C>	As<B>	As<S>	Bars<C>	Bars<B>	Bars<S>
1	350	350	0	1000	1000	0	2-#25	2-#25	---
2	--- Not checked ---								
3	--- Not checked ---								
4	350	350	0	1000	1000	0	2-#25	2-#25	---

<C> Total Strip, <B> Banded Strip, <S> Remaining Strip

Bottom Reinforcement

Units: Width (m), Mmax (kNm), Xmax (m), As (mm^2), Sp (mm)

Span	Strip	Width	Mmax	Xmax	AsMin	AsMax	AsReq	SpProv	Bars
1	Column	2.40	0.00	0.093	0	6835	0	0	---
	Middle	3.75	0.00	0.093	0	10680	0	0	---
	Beam	0.35	0.00	0.093	0	3590	0	0	---
2	Column	2.40	20.30	2.453	744	6835	480	400	6-#15 *3 *5
	Middle	3.75	31.72	2.453	1162	10680	750	417	9-#15 *3 *5
	Beam	0.35	64.33	2.453	438	3590	428	220	2-#25 *3
3	Column	2.40	15.48	2.750	744	6835	364	400	6-#15 *3 *5
	Middle	3.75	24.19	2.750	1162	10680	569	417	9-#15 *3 *5
	Beam	0.35	49.05	2.750	438	3590	324	220	2-#25 *3
4	Column	2.40	20.30	3.047	744	6835	480	400	6-#15 *3 *5
	Middle	3.75	31.72	3.047	1162	10680	750	417	9-#15 *3 *5
	Beam	0.35	64.33	3.047	438	3590	428	220	2-#25 *3
5	Column	2.40	0.00	0.132	0	6835	0	0	---
	Middle	3.75	0.00	0.132	0	10680	0	0	---
	Beam	0.35	0.00	0.132	0	3590	0	0	---

NOTES:
*3 - Design governed by minimum reinforcement.
*5 - Number of bars governed by maximum allowable spacing.

Bottom Bar Details

Units: Start (m), Length (m)

		Long Bars			Short Bars		
Span	Strip	Bars	Start	Length	Bars	Start	Length
1	Column	---			---		
	Middle	---			---		
	Beam	---			---		
2	Column	6-#15	0.00	5.50	---		
	Middle	6-#15	0.00	5.50	3-#15	0.00	4.67
	Beam	2-#25	0.00	5.50	---		
3	Column	6-#15	0.00	5.50	---		
	Middle	6-#15	0.00	5.50	3-#15	0.82	3.85
	Beam	2-#25	0.00	5.50	---		
4	Column	6-#15	0.00	5.50	---		
	Middle	6-#15	0.00	5.50	3-#15	0.82	4.68
	Beam	2-#25	0.00	5.50	---		
5	Column	---			---		
	Middle	---			---		
	Beam	---			---		

Bottom Bar Development Lengths

Units: DevLen (mm)

		Long Bars		Short Bars	
Span	Strip	Bars	DevLen	Bars	DevLen
1	Column	---		---	
	Middle	---		---	

	Beam	---		---	
2	Column	6-#15	300.00	---	
	Middle	6-#15	300.00	3-#15	300.00
	Beam	2-#25	461.45	---	
3	Column	6-#15	300.00	---	
	Middle	6-#15	300.00	3-#15	300.00
	Beam	2-#25	348.85	---	
4	Column	6-#15	300.00	---	
	Middle	6-#15	300.00	3-#15	300.00
	Beam	2-#25	461.45	---	
5	Column	---		---	
	Middle	---		---	
	Beam	---		---	

Flexural Capacity
================

Units: x (m), As (mm^2), PhiMn, Mu (kNm)

			Top						Bottom					
Span	Strip	x	AsTop	PhiMn-	Mu-	Comb	Pat	Status	AsBot	PhiMn+	Mu+	Comb	Pat	Status
1	Column	0.000	1200	-49.19	0.00	U1	All	OK	0	0.00	0.00	U1	All	OK
		0.065	1200	-49.19	-0.00	U1	All	OK	0	0.00	0.00	U1	All	OK
		0.113	1200	-49.19	-0.00	U1	All	OK	0	0.00	0.00	U1	All	OK
		0.121	1200	-49.19	-0.00	U1	All	OK	0	0.00	0.00	U1	All	OK
		0.186	1200	-49.19	-0.00	U1	All	OK	0	0.00	0.00	U1	All	OK
		0.225	1200	-49.19	-0.00	U1	All	---	0	0.00	0.00	U1	All	---
	Middle	0.000	1800	-73.94	0.00	U1	All	OK	0	0.00	0.00	U1	All	OK
		0.065	1800	-73.94	-0.00	U1	All	OK	0	0.00	0.00	U1	All	OK
		0.113	1800	-73.94	-0.00	U1	All	OK	0	0.00	0.00	U1	All	OK
		0.121	1800	-73.94	-0.00	U1	All	OK	0	0.00	0.00	U1	All	OK
		0.186	1800	-73.94	-0.00	U1	All	OK	0	0.00	0.00	U1	All	OK
		0.225	1800	-73.94	-0.00	U1	All	---	0	0.00	0.00	U1	All	---
	Beam	0.000	1000	-143.01	0.00	U1	All	OK	0	0.00	0.00	U1	All	OK
		0.065	1000	-143.01	-0.20	U1	All	OK	0	0.00	0.00	U1	All	OK
		0.113	1000	-143.01	-0.57	U1	All	OK	0	0.00	0.00	U1	All	OK
		0.121	1000	-143.01	-0.64	U1	All	OK	0	0.00	0.00	U1	All	OK
		0.186	1000	-143.01	-1.45	U1	All	OK	0	0.00	0.00	U1	All	OK
		0.225	1000	-143.01	-2.04	U1	All	---	0	0.00	0.00	U1	All	---
2	Column	0.000	1200	-49.19	1.04	U1	All	---	1200	49.19	0.00	U1	All	---
		0.225	1200	-49.19	-0.00	U1	All	OK	1200	49.19	0.00	U1	All	OK
		0.241	1179	-48.38	-0.05	U1	All	OK	1200	49.19	0.00	U1	All	OK
		0.522	53	-2.30	-0.43	U1	Even	OK	1200	49.19	0.00	U1	All	OK
		0.525	42	-1.80	-0.43	U1	Even	OK	1200	49.19	0.00	U1	All	OK
		0.541	0	0.00	-0.43	U1	Even	*EXCEEDED	1200	49.19	0.00	U1	All	OK
		0.819	0	0.00	-0.04	U1	Even	*EXCEEDED	1200	49.19	1.21	U1	Odd	OK
		1.993	0	0.00	0.00	U1	All	OK	1200	49.19	18.57	U1	All	OK
		2.453	0	0.00	0.00	U1	All	OK	1200	49.19	20.30	U1	All	OK
		2.750	0	0.00	0.00	U1	All	OK	1200	49.19	19.84	U1	All	OK
		3.474	0	0.00	0.00	U1	All	OK	1200	49.19	14.48	U1	Even	OK
		3.508	74	-3.18	0.00	U1	All	OK	1200	49.19	14.12	U1	Even	OK
		3.836	800	-33.38	-0.22	U1	Odd	OK	1200	49.19	9.92	U1	Even	OK
		4.913	800	-33.38	-19.50	U1	All	OK	1200	49.19	0.00	U1	All	OK
		5.275	1200	-49.19	-34.27	U1	All	OK	1200	49.19	0.00	U1	All	OK
		5.500	1200	-49.19	-45.38	U1	All	---	1200	49.19	0.00	U1	All	---
	Middle	0.000	1800	-73.94	1.62	U1	All	---	1800	73.94	0.00	U1	All	---
		0.225	1800	-73.94	-0.00	U1	All	OK	1800	73.94	0.00	U1	All	OK
		0.241	1800	-73.94	-0.07	U1	All	OK	1800	73.94	0.00	U1	All	OK
		0.522	111	-4.80	-0.67	U1	Even	OK	1800	73.94	0.00	U1	All	OK
		0.541	0	0.00	-0.67	U1	Even	*EXCEEDED	1800	73.94	0.00	U1	All	OK
		0.819	0	0.00	-0.06	U1	Even	*EXCEEDED	1800	73.94	1.89	U1	Odd	OK
		1.993	0	0.00	0.00	U1	All	OK	1800	73.94	29.01	U1	All	OK
		2.453	0	0.00	0.00	U1	All	OK	1800	73.94	31.72	U1	All	OK
		2.750	0	0.00	0.00	U1	All	OK	1800	73.94	30.99	U1	All	OK
		3.474	0	0.00	0.00	U1	All	OK	1800	73.94	22.62	U1	Even	OK
		3.508	159	-6.86	0.00	U1	All	OK	1800	73.94	22.07	U1	Even	OK
		3.851	1800	-73.94	-0.50	U1	Odd	OK	1800	73.94	15.16	U1	Even	OK
		4.375	1800	-73.94	-8.00	U1	Odd	OK	1800	73.94	0.37	U1	Even	OK
		4.675	1800	-73.94	-18.30	U1	All	OK	1200	50.13	0.00	U1	All	OK
		5.275	1800	-73.94	-53.55	U1	All	OK	1200	50.13	0.00	U1	All	OK
		5.500	1800	-73.94	-70.91	U1	All	---	1200	50.13	0.00	U1	All	---
	Beam	0.000	1000	-143.01	-136.22	U1	All	---	1000	143.01	0.00	U1	All	---
		0.225	1000	-143.01	-90.42	U1	All	OK	1000	143.01	0.00	U1	All	OK
		0.692	645	-95.16	-17.54	U1	Even	OK	1000	143.01	0.28	U1	Odd	OK
		0.884	355	-53.57	0.00	U1	All	OK	1000	143.01	6.84	U1	All	OK
		1.351	0	0.00	0.00	U1	All	OK	1000	143.01	35.42	U1	All	OK
		1.993	0	0.00	0.00	U1	All	OK	1000	143.01	58.83	U1	All	OK
		2.453	0	0.00	0.00	U1	All	OK	1000	143.01	64.33	U1	All	OK
		2.750	0	0.00	0.00	U1	All	OK	1000	143.01	62.85	U1	All	OK
		3.332	0	0.00	0.00	U1	All	OK	1000	143.01	50.15	U1	Even	OK
		3.508	110	-16.88	0.00	U1	All	OK	1000	143.01	44.75	U1	Even	OK
		4.132	500	-74.63	-12.55	U1	Odd	OK	1000	143.01	15.95	U1	Even	OK
		4.475	500	-74.63	-26.88	U1	Odd	OK	1000	143.01	0.00	U1	All	OK

		5.275	1000	-143.01	-108.59	U1	All	OK	1000	143.01	0.00	U1	All	OK
		5.500	1000	-143.01	-132.70	U1	All	---	1000	143.01	0.00	U1	All	---
3	Column	0.000	1200	-49.19	-39.48	U1	All	---	1200	49.19	0.00	U1	All	---
		0.225	1200	-49.19	-31.13	U1	All	OK	1200	49.19	0.00	U1	All	OK
		0.553	800	-33.38	-20.24	U1	All	OK	1200	49.19	0.00	U1	All	OK
		1.921	800	-33.38	-0.01	U1	Even	OK	1200	49.19	11.34	U1	Odd	OK
		1.993	625	-26.29	0.00	U1	All	OK	1200	49.19	12.03	U1	Odd	OK
		2.249	0	0.00	0.00	U1	All	OK	1200	49.19	13.96	U1	Odd	OK
		2.750	0	0.00	0.00	U1	All	OK	1200	49.19	15.48	U1	Odd	OK
		3.251	0	0.00	0.00	U1	All	OK	1200	49.19	13.96	U1	Odd	OK
		3.508	625	-26.29	0.00	U1	All	OK	1200	49.19	12.03	U1	Odd	OK
		3.579	800	-33.38	-0.01	U1	Even	OK	1200	49.19	11.34	U1	Odd	OK
		4.947	800	-33.38	-20.24	U1	All	OK	1200	49.19	0.00	U1	All	OK
		5.275	1200	-49.19	-31.13	U1	All	OK	1200	49.19	0.00	U1	All	OK
		5.500	1200	-49.19	-39.48	U1	All	---	1200	49.19	0.00	U1	All	---
	Middle	0.000	1800	-73.94	-61.68	U1	All	---	1200	50.13	0.00	U1	All	---
		0.225	1800	-73.94	-48.64	U1	All	OK	1200	50.13	0.00	U1	All	OK
		0.825	1800	-73.94	-19.29	U1	All	OK	1200	50.13	0.00	U1	All	OK
		1.125	1800	-73.94	-10.21	U1	S1	OK	1800	73.94	0.55	U1	S3	OK
		1.907	1800	-73.94	-0.12	U1	Even	OK	1800	73.94	17.50	U1	Odd	OK
		1.993	1351	-56.19	0.00	U1	All	OK	1800	73.94	18.79	U1	Odd	OK
		2.249	0	0.00	0.00	U1	All	OK	1800	73.94	21.82	U1	Odd	OK
		2.750	0	0.00	0.00	U1	All	OK	1800	73.94	24.19	U1	Odd	OK
		3.251	0	0.00	0.00	U1	All	OK	1800	73.94	21.82	U1	Odd	OK
		3.508	1351	-56.19	0.00	U1	All	OK	1800	73.94	18.79	U1	Odd	OK
		3.593	1800	-73.94	-0.12	U1	Even	OK	1800	73.94	17.50	U1	Odd	OK
		4.375	1800	-73.94	-10.21	U1	S4	OK	1800	73.94	0.55	U1	S2	OK
		4.675	1800	-73.94	-19.29	U1	All	OK	1200	50.13	0.00	U1	All	OK
		5.275	1800	-73.94	-48.64	U1	All	OK	1200	50.13	0.00	U1	All	OK
		5.500	1800	-73.94	-61.68	U1	All	---	1200	50.13	0.00	U1	All	---
	Beam	0.000	1000	-143.01	-125.08	U1	All	---	1000	143.01	0.00	U1	All	---
		0.225	1000	-143.01	-98.62	U1	All	OK	1000	143.01	0.00	U1	All	OK
		0.947	500	-74.63	-28.98	U1	All	OK	1000	143.01	0.00	U1	All	OK
		1.668	500	-74.63	-4.55	U1	Even	OK	1000	143.01	26.74	U1	Odd	OK
		1.993	276	-41.91	0.00	U1	All	OK	1000	143.01	38.10	U1	Odd	OK
		2.390	0	0.00	0.00	U1	All	OK	1000	143.01	46.58	U1	Odd	OK
		2.750	0	0.00	0.00	U1	All	OK	1000	143.01	49.05	U1	Odd	OK
		3.110	0	0.00	0.00	U1	All	OK	1000	143.01	46.58	U1	Odd	OK
		3.508	276	-41.91	0.00	U1	All	OK	1000	143.01	38.10	U1	Odd	OK
		3.832	500	-74.63	-4.55	U1	Even	OK	1000	143.01	26.74	U1	Odd	OK
		4.553	500	-74.63	-28.98	U1	All	OK	1000	143.01	0.00	U1	All	OK
		5.275	1000	-143.01	-98.62	U1	All	OK	1000	143.01	0.00	U1	All	OK
		5.500	1000	-143.01	-125.08	U1	All	---	1000	143.01	0.00	U1	All	---
4	Column	0.000	1200	-49.19	-45.38	U1	All	---	1200	49.19	0.00	U1	All	---
		0.225	1200	-49.19	-34.27	U1	All	OK	1200	49.19	0.00	U1	All	OK
		0.587	800	-33.38	-19.50	U1	All	OK	1200	49.19	0.00	U1	All	OK
		1.664	800	-33.38	-0.22	U1	Odd	OK	1200	49.19	9.92	U1	Even	OK
		1.993	74	-3.18	0.00	U1	All	OK	1200	49.19	14.12	U1	Even	OK
		2.026	0	0.00	0.00	U1	All	OK	1200	49.19	14.48	U1	Even	OK
		2.750	0	0.00	0.00	U1	All	OK	1200	49.19	19.84	U1	All	OK
		3.047	0	0.00	0.00	U1	All	OK	1200	49.19	20.30	U1	All	OK
		3.508	0	0.00	0.00	U1	All	OK	1200	49.19	18.57	U1	All	OK
		4.681	0	0.00	-0.04	U1	Even	*EXCEEDED	1200	49.19	1.21	U1	Odd	OK
		4.959	0	0.00	-0.43	U1	Even	*EXCEEDED	1200	49.19	0.00	U1	All	OK
		4.975	42	-1.80	-0.43	U1	Even	OK	1200	49.19	0.00	U1	All	OK
		4.978	53	-2.30	-0.43	U1	Even	OK	1200	49.19	0.00	U1	All	OK
		5.259	1179	-48.38	-0.05	U1	All	OK	1200	49.19	0.00	U1	All	OK
		5.275	1200	-49.19	-0.00	U1	All	OK	1200	49.19	0.00	U1	All	OK
		5.500	1200	-49.19	1.04	U1	All	---	1200	49.19	0.00	U1	All	---
	Middle	0.000	1800	-73.94	-70.91	U1	All	---	1200	50.13	0.00	U1	All	---
		0.225	1800	-73.94	-53.55	U1	All	OK	1200	50.13	0.00	U1	All	OK
		0.825	1800	-73.94	-18.30	U1	All	OK	1200	50.13	0.00	U1	All	OK
		1.125	1800	-73.94	-8.00	U1	Odd	OK	1800	73.94	0.37	U1	Even	OK
		1.649	1800	-73.94	-0.50	U1	Odd	OK	1800	73.94	15.16	U1	Even	OK
		1.993	159	-6.86	0.00	U1	All	OK	1800	73.94	22.07	U1	Even	OK
		2.026	0	0.00	0.00	U1	All	OK	1800	73.94	22.62	U1	Even	OK
		2.750	0	0.00	0.00	U1	All	OK	1800	73.94	30.99	U1	All	OK
		3.047	0	0.00	0.00	U1	All	OK	1800	73.94	31.72	U1	All	OK
		3.508	0	0.00	0.00	U1	All	OK	1800	73.94	29.01	U1	All	OK
		4.681	0	0.00	-0.06	U1	Even	*EXCEEDED	1800	73.94	1.89	U1	Odd	OK
		4.959	0	0.00	-0.67	U1	Even	*EXCEEDED	1800	73.94	0.00	U1	All	OK
		4.978	111	-4.80	-0.67	U1	Even	OK	1800	73.94	0.00	U1	All	OK
		5.259	1800	-73.94	-0.07	U1	All	OK	1800	73.94	0.00	U1	All	OK
		5.275	1800	-73.94	-0.00	U1	All	OK	1800	73.94	0.00	U1	All	OK
		5.500	1800	-73.94	1.62	U1	All	---	1800	73.94	0.00	U1	All	---
	Beam	0.000	1000	-143.01	-132.70	U1	All	---	1000	143.01	0.00	U1	All	---
		0.225	1000	-143.01	-108.59	U1	All	OK	1000	143.01	0.00	U1	All	OK
		1.025	500	-74.63	-26.88	U1	Odd	OK	1000	143.01	0.00	U1	All	OK
		1.368	500	-74.63	-12.55	U1	Odd	OK	1000	143.01	15.95	U1	Even	OK
		1.993	110	-16.88	0.00	U1	All	OK	1000	143.01	44.75	U1	Even	OK
		2.168	0	0.00	0.00	U1	All	OK	1000	143.01	50.15	U1	Even	OK
		2.750	0	0.00	0.00	U1	All	OK	1000	143.01	62.85	U1	All	OK
		3.047	0	0.00	0.00	U1	All	OK	1000	143.01	64.33	U1	All	OK
		3.508	0	0.00	0.00	U1	All	OK	1000	143.01	58.83	U1	All	OK
		4.149	0	0.00	0.00	U1	All	OK	1000	143.01	35.42	U1	All	OK

	4.616	355	-53.57	0.00	U1	All	OK	1000	143.01	6.84	U1	All	OK
	4.808	645	-95.16	-17.54	U1	Even	OK	1000	143.01	0.28	U1	Odd	OK
	5.275	1000	-143.01	-90.42	U1	All	OK	1000	143.01	0.00	U1	All	OK
	5.500	1000	-143.01	-136.22	U1	All	---	1000	143.01	0.00	U1	All	---
5 Column	0.000	1200	-49.19	-0.00	U1	All	---	0	0.00	0.00	U1	All	---
	0.039	1200	-49.19	-0.00	U1	All	OK	0	0.00	0.00	U1	All	OK
	0.104	1200	-49.19	-0.00	U1	All	OK	0	0.00	0.00	U1	All	OK
	0.113	1200	-49.19	-0.00	U1	All	OK	0	0.00	0.00	U1	All	OK
	0.160	1200	-49.19	-0.00	U1	All	OK	0	0.00	0.00	U1	All	OK
	0.225	1200	-49.19	-0.00	U1	All	OK	0	0.00	0.00	U1	All	OK
Middle	0.000	1800	-73.94	-0.00	U1	All	---	0	0.00	0.00	U1	All	---
	0.039	1800	-73.94	-0.00	U1	All	OK	0	0.00	0.00	U1	All	OK
	0.104	1800	-73.94	-0.00	U1	All	OK	0	0.00	0.00	U1	All	OK
	0.113	1800	-73.94	-0.00	U1	All	OK	0	0.00	0.00	U1	All	OK
	0.160	1800	-73.94	-0.00	U1	All	OK	0	0.00	0.00	U1	All	OK
	0.225	1800	-73.94	-0.00	U1	All	OK	0	0.00	0.00	U1	All	OK
Beam	0.000	1000	-143.01	-2.04	U1	All	---	0	0.00	0.00	U1	All	---
	0.039	1000	-143.01	-1.45	U1	All	OK	0	0.00	0.00	U1	All	OK
	0.104	1000	-143.01	-0.64	U1	All	OK	0	0.00	0.00	U1	All	OK
	0.113	1000	-143.01	-0.57	U1	All	OK	0	0.00	0.00	U1	All	OK
	0.160	1000	-143.01	-0.20	U1	All	OK	0	0.00	0.00	U1	All	OK
	0.225	1000	-143.01	-0.00	U1	All	OK	0	0.00	0.00	U1	All	OK

Longitudinal Beam Transverse Reinforcement Demand and Capacity

Section Properties

Units: dv (mm), Av/s (mm^2/mm), PhiVc, Vrmax (kN)

Span	dv	(Av/s)min	PhiVc	Vrmax
1	411.7	0.263	84.29	585.33
2	411.7	0.263	84.29	585.33
3	411.7	0.263	84.29	585.33
4	411.7	0.263	84.29	585.33
5	411.7	0.263	84.29	585.33

Beam Transverse Reinforcement Demand

Units: Start, End, Xu (mm), Vu (m), Av/s (kN/mm^2)

			Required				Demand	
Span	Start	End	Xu	Vu	Comb/Patt	Av/s	Av/s	
1	0.000	0.000	0.000	0.00	U1/All	0.000	0.000	
2	0.301	1.240	0.637	149.45	U1/All	0.326	0.326	
	1.240	1.844	1.240	100.76	U1/All	0.082	0.263	*8
	1.844	2.448	1.844	52.86	U1/Even	0.000	0.000	
	2.448	3.052	3.052	45.33	U1/All	0.000	0.000	
	3.052	3.656	3.656	94.03	U1/All	0.049	0.263	*8
	3.656	4.260	4.260	142.73	U1/All	0.292	0.292	
	4.260	5.199	4.863	191.42	U1/All	0.536	0.536	
3	0.301	1.240	0.637	170.44	U1/All	0.431	0.431	
	1.240	1.844	1.240	121.74	U1/All	0.187	0.263	*8
	1.844	2.448	1.844	73.04	U1/All	0.000	0.000	
	2.448	3.052	2.448	30.11	U1/S2	0.000	0.000	
	3.052	3.656	3.656	73.04	U1/All	0.000	0.000	
	3.656	4.260	4.260	121.74	U1/All	0.187	0.263	*8
	4.260	5.199	4.863	170.44	U1/All	0.431	0.431	
4	0.301	1.240	0.637	191.42	U1/All	0.536	0.536	
	1.240	1.844	1.240	142.73	U1/All	0.292	0.292	
	1.844	2.448	1.844	94.03	U1/All	0.049	0.263	*8
	2.448	3.052	2.448	45.33	U1/All	0.000	0.000	
	3.052	3.656	3.656	52.86	U1/Even	0.000	0.000	
	3.656	4.260	4.260	100.76	U1/All	0.082	0.263	*8
	4.260	5.199	4.863	149.45	U1/All	0.326	0.326	
5	0.225	0.225	0.225	0.00	U1/All	0.000	0.000	

NOTES:
*8 - Minimum transverse (stirrup) reinforcement governs.

Beam Transverse Reinforcement Details

Units: spacing & distance (mm).

Span	Size	Stirrups (2 legs each unless otherwise noted)
1	#10	--- None ---
2	#10	6 @ 281 + <-- 1208 --> + 8 @ 286
3	#10	6 @ 281 + <-- 1811 --> + 6 @ 281
4	#10	8 @ 286 + <-- 1208 --> + 6 @ 281
5	#10	--- None ---

Beam Transverse Reinforcement Capacity

Units: Start, End, Xu (m), Vu, PhiVn (kN), Av/s (mm^2/mm), Av (mm^2), Sp (mm)

			Required					Provided			
Span	Start	End	Xu	Vu	Comb/Patt	Av/s	Reqd/Min	Av	Sp	Av/s	PhiVn
1	0.000	0.225	0.000	0.00	U1/All	-----	-----	-----	-----	-----	-----
2	0.000	0.301	0.637	149.45	U1/All	-----	-----	-----	-----	-----	-----
	0.301	1.844	0.637	149.45	U1/All	0.326	1.24	200.0	281	0.713	226.78
	1.844	3.052	1.844	52.86	U1/Even	0.000	0.00	-----	-----	-----	76.29
	3.052	5.199	4.863	191.42	U1/All	0.536	2.04	200.0	286	0.699	223.95
	5.199	5.500	4.863	191.42	U1/All	-----	-----	-----	-----	-----	-----
3	0.000	0.301	0.637	170.44	U1/All	-----	-----	-----	-----	-----	-----
	0.301	1.844	0.637	170.44	U1/All	0.431	1.64	200.0	281	0.713	226.78
	1.844	3.656	1.844	73.04	U1/All	0.000	0.00	-----	-----	-----	76.29
	3.656	5.199	4.863	170.44	U1/All	0.431	1.64	200.0	281	0.713	226.78
	5.199	5.500	4.863	170.44	U1/All	-----	-----	-----	-----	-----	-----
4	0.000	0.301	0.637	191.42	U1/All	-----	-----	-----	-----	-----	-----
	0.301	2.448	0.637	191.42	U1/All	0.536	2.04	200.0	286	0.699	223.95
	2.448	3.656	3.656	52.86	U1/Even	0.000	0.00	-----	-----	-----	76.29
	3.656	5.199	4.863	149.45	U1/All	0.326	1.24	200.0	281	0.713	226.78
	5.199	5.500	4.863	149.45	U1/All	-----	-----	-----	-----	-----	-----
5	0.000	0.225	0.225	0.00	U1/All	-----	-----	-----	-----	-----	-----

Slab Shear Capacity

Units: b, dv (mm), Xu (m), PhiVc, Vu(kN)

Span	b	dv	Beta	Vratio	PhiVc	Vu	Xu
1	6150	114	0.210	0.000	479.76	0.00	0.00
2	6150	114	0.210	0.000	479.76	0.00	5.16
3	6150	114	0.210	0.000	479.76	0.00	0.34
4	6150	114	0.210	0.000	479.76	0.00	0.34
5	6150	114	0.210	0.000	479.76	0.00	0.00

Flexural Transfer of Negative Unbalanced Moment at Supports

Units: Width (mm), Munb (kNm), As (mm^2)

Supp	Width	Width-c	d	Munb	Comb	Pat	GammaF	AsReq	AsProv	Add Bars
1	915	915	117	131.52	U1	All	0.614	3519	1283	12-#15
2	915	915	117	62.68	U1	Even	0.600	1088	1283	---
3	915	915	117	62.68	U1	Even	0.600	1088	1283	---
4	915	915	117	131.52	U1	All	0.614	3519	1283	12-#15

Punching Shear Around Columns

Critical Section Properties

Units: b1, b2, b0, davg, CG, c(left), c(right) (mm), Ac (mm^2), Jc (mm^4)

Supp	Type	b1	b2	b0	davg	CG	c(left)	c(right)	Ac	Jc
1	Rect	513.5	577.0	1604.0	440.1	58.1	283.1	230.4	7.0596e+005	3.9366e+010
2	Rect	577.0	577.0	2308.0	336.3	0.0	288.5	288.5	7.7612e+005	4.5042e+010
3	Rect	577.0	577.0	2308.0	336.3	0.0	288.5	288.5	7.7612e+005	4.5042e+010
4	Rect	513.5	577.0	1604.0	440.1	-58.1	230.4	283.1	7.0596e+005	3.9366e+010

Punching Shear Results

Units: Vu (kN), Munb (kNm), vu (N/mm^2), Phi*vc (N/mm^2)

Supp	Vu	vu	Munb	Comb	Pat	GammaV	vu	Phi*vc
1	215.27	0.305	119.00	U1	All	0.386	0.574	1.115
2	460.42	0.593	-22.74	U1	All	0.400	0.652	1.201
3	460.42	0.593	22.74	U1	All	0.400	0.652	1.201
4	215.27	0.305	-119.00	U1	All	0.386	0.574	1.115

Integrity Reinforcement at Supports

Units: Vse(kN), Asb(mm^2)

Supp	Vse	Asb
1	149.92	750 #
2	327.64	1638 #
3	327.64	1638 #
4	149.92	750 #

#Beams present. Integrity reinforcement may not be required.
NOTES: The sum of bottom reinforcement crossing the perimeter of the support on all sides shall not be less than the above listed values.

Material Takeoff

Reinforcement in the Direction of Analysis

Top Bars: 289.9 kg <=> 17.11 kg/m <=> 2.632 kg/m^2

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   Bottom Bars:   502.6 kg   <=>  29.65 kg/m    <=>  4.561 kg/m^2
   Stirrups:       46.1 kg   <=>   2.72 kg/m    <=>  0.418 kg/m^2
   Total Steel:   838.6 kg   <=>  49.47 kg/m    <=>  7.612 kg/m^2
   Concrete:       23.2 m^3  <=>   1.37 m^3/m   <=>  0.210 m^3/m^2
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spSlab v5.00 (TM)
A Computer Program for Analysis, Design, and Investigation of
Reinforced Concrete Beams, One-way and Two-way Slab Systems

[3] DEFLECTION RESULTS

Section Properties

Frame Section Properties

Units: Ig, Icr (mm^4), Mcr (kNm)

Span	Zone	M+ve Ig	M+ve Icr	M+ve Mcr	M-ve Ig	M-ve Icr	M-ve Mcr
1	Left	9.9540e+009	0.00000	37.73	3.6458e+009	3.1596e+009	-21.88
	Midspan	9.9540e+009	0.00000	37.73	3.6458e+009	3.1596e+009	-21.88
	Right	9.9540e+009	0.00000	37.73	3.6458e+009	3.1596e+009	-21.88
2	Left	9.9540e+009	1.0881e+009	37.73	3.6458e+009	3.1596e+009	-21.88
	Midspan	9.9540e+009	1.6364e+009	37.73	3.6458e+009	0.00000	-21.88
	Right	9.9540e+009	1.0881e+009	37.73	3.6458e+009	3.1596e+009	-21.88
3	Left	9.9540e+009	1.0881e+009	37.73	3.6458e+009	3.1596e+009	-21.88
	Midspan	9.9540e+009	1.6364e+009	37.73	3.6458e+009	0.00000	-21.88
	Right	9.9540e+009	1.0881e+009	37.73	3.6458e+009	3.1596e+009	-21.88
4	Left	9.9540e+009	1.0881e+009	37.73	3.6458e+009	3.1596e+009	-21.88
	Midspan	9.9540e+009	1.6364e+009	37.73	3.6458e+009	0.00000	-21.88
	Right	9.9540e+009	1.0881e+009	37.73	3.6458e+009	3.1596e+009	-21.88
5	Left	9.9540e+009	0.00000	37.73	3.6458e+009	3.1596e+009	-21.88
	Midspan	9.9540e+009	0.00000	37.73	3.6458e+009	3.1596e+009	-21.88
	Right	9.9540e+009	0.00000	37.73	3.6458e+009	3.1596e+009	-21.88

NOTES: M+ve values are for positive moments (tension at bottom face).
M-ve values are for negative moments (tension at top face).

Frame Effective Section Properties

Units: Ie, Ie,avg (mm^4), Mmax (kNm)

			Load Level					
			Dead		Sustained		Dead+Live	
Span	Zone	Weight	Mmax	Ie	Mmax	Ie	Mmax	Ie
1	Right	1.000	-0.69	3.6458e+009	-0.69	3.6458e+009	-1.48	3.6458e+009
	Span Avg	----	----	3.6458e+009	----	3.6458e+009	----	3.6458e+009
2	Middle	0.850	39.07	9.1277e+009	39.07	9.1277e+009	84.08	2.3879e+009
	Right	0.150	-83.60	3.1683e+009	-83.60	3.1683e+009	-179.92	3.1605e+009
	Span Avg	----	----	8.2338e+009	----	8.2338e+009	----	2.5038e+009
3	Left	0.150	-75.96	3.1712e+009	-75.96	3.1712e+009	-163.49	3.1608e+009
	Middle	0.700	26.43	9.9540e+009	26.43	9.9540e+009	56.88	4.0641e+009
	Right	0.150	-75.96	3.1712e+009	-75.96	3.1712e+009	-163.49	3.1608e+009
	Span Avg	----	----	7.9191e+009	----	7.9191e+009	----	3.7931e+009
4	Left	0.150	-83.60	3.1683e+009	-83.60	3.1683e+009	-179.92	3.1605e+009
	Middle	0.850	39.07	9.1277e+009	39.07	9.1277e+009	84.08	2.3879e+009
	Span Avg	----	----	8.2338e+009	----	8.2338e+009	----	2.5038e+009
5	Left	1.000	-0.69	3.6458e+009	-0.69	3.6458e+009	-1.48	3.6458e+009
	Span Avg	----	----	3.6458e+009	----	3.6458e+009	----	3.6458e+009

Strip Section Properties at Midspan

Units: Ig (mm^4)

Span	Column Strip Ig	Column Strip LDF	Column Strip Ratio	Middle Strip Ig	Middle Strip LDF	Middle Strip Ratio
1	7.93198e+009	0.896	1.125	1.16371e+009	0.104	0.886
2	7.93198e+009	0.796	0.998	1.16371e+009	0.204	1.749
3	7.93198e+009	0.727	0.913	1.16371e+009	0.273	2.332
4	7.93198e+009	0.796	0.998	1.16371e+009	0.204	1.749
5	7.93198e+009	0.896	1.125	1.16371e+009	0.104	0.886

NOTES: Load distirubtion factor, LDL, averages moment distribution factors listed in [2] Design Results.
Ratio refers to proportion of strip to frame deflections under fix-end condtions.

Instantaneous Deflections

Extreme Instantaneous Frame Deflections and Corresponding Locations

Units: Def (mm), Loc (m)

Span	Direction	Value	Dead	Live Sustained	Live Unsustained	Live Total	Total Sustained	Total Dead+Live
1	Down	Def	---	---	---	---	---	---
		Loc	---	---	---	---	---	---
	Up	Def	-0.03	---	-0.06	-0.06	-0.03	-0.10
		Loc	0.000	---	0.000	0.000	0.000	0.000
2	Down	Def	0.37	---	1.55	1.55	0.37	1.91
		Loc	2.527	---	2.676	2.676	2.527	2.601
	Up	Def	---	---	---	---	---	---
		Loc	---	---	---	---	---	---
3	Down	Def	0.19	---	0.78	0.78	0.19	0.97
		Loc	2.750	---	2.750	2.750	2.750	2.750
	Up	Def	-0.01	---	-0.00	-0.00	-0.01	-0.01
		Loc	0.299	---	0.225	0.225	0.299	0.225
4	Down	Def	0.37	---	1.55	1.55	0.37	1.91
		Loc	2.973	---	2.824	2.824	2.973	2.899
	Up	Def	---	---	---	---	---	---
		Loc	---	---	---	---	---	---
5	Down	Def	---	---	---	---	---	---
		Loc	---	---	---	---	---	---
	Up	Def	-0.03	---	-0.06	-0.06	-0.03	-0.10
		Loc	0.225	---	0.225	0.225	0.225	0.225

Extreme Instantaneous Column Strip Deflections and Corresponding Locations

Units: Def (mm), Loc (m)

Span	Direction	Value	Dead	Live Sustained	Live Unsustained	Live Total	Total Sustained	Total Dead+Live
1	Down	Def	---	---	---	---	---	---
		Loc	---	---	---	---	---	---
	Up	Def	-0.03	---	-0.06	-0.06	-0.03	-0.10
		Loc	0.000	---	0.000	0.000	0.000	0.000
2	Down	Def	0.36	---	1.54	1.54	0.36	1.91
		Loc	2.527	---	2.676	2.676	2.527	2.601
	Up	Def	---	---	---	---	---	---
		Loc	---	---	---	---	---	---
3	Down	Def	0.17	---	0.71	0.71	0.17	0.88
		Loc	2.750	---	2.750	2.750	2.750	2.750
	Up	Def	-0.01	---	-0.00	-0.00	-0.01	-0.01
		Loc	0.299	---	0.225	0.225	0.299	0.225
4	Down	Def	0.36	---	1.54	1.54	0.36	1.91
		Loc	2.973	---	2.824	2.824	2.973	2.899
	Up	Def	---	---	---	---	---	---
		Loc	---	---	---	---	---	---
5	Down	Def	---	---	---	---	---	---
		Loc	---	---	---	---	---	---
	Up	Def	-0.03	---	-0.06	-0.06	-0.03	-0.10
		Loc	0.225	---	0.225	0.225	0.225	0.225

Extreme Instantaneous Middle Strip Deflections and Corresponding Locations

Units: Def (mm), Loc (m)

Span	Direction	Value	Dead	Live Sustained	Live Unsustained	Live Total	Total Sustained	Total Dead+Live
1	Down	Def	---	---	---	---	---	---
		Loc	---	---	---	---	---	---
	Up	Def	-0.03	---	-0.06	-0.06	-0.03	-0.10
		Loc	0.000	---	0.000	0.000	0.000	0.000
2	Down	Def	0.53	---	2.54	2.54	0.53	3.07
		Loc	2.601	---	2.676	2.676	2.601	2.676
	Up	Def	---	---	---	---	---	---
		Loc	---	---	---	---	---	---
3	Down	Def	0.50	---	1.85	1.85	0.50	2.35
		Loc	2.750	---	2.750	2.750	2.750	2.750
	Up	Def	-0.00	---	-0.00	-0.00	-0.00	-0.00
		Loc	0.225	---	0.112	0.112	0.225	0.225

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4	Down	Def	0.53	---	2.54	2.54	0.53	3.07
		Loc	2.899	---	2.824	2.824	2.899	2.824
	Up	Def	---	---	---	---	---	---
		Loc	---	---	---	---	---	---
5	Down	Def	---	---	---	---	---	---
		Loc	---	---	---	---	---	---
	Up	Def	-0.03	---	-0.06	-0.06	-0.03	-0.10
		Loc	0.225	---	0.225	0.225	0.225	0.225

Long-term Deflections

Long-term Column Strip Deflection Factors

Time dependant factor for sustained loads = 2.000
Units: Astop, Asbot (mm^2), b, d (mm), Rho' (%), Lambda (-)

		M+ve					M-ve				
Span	Zone	Astop	b	d	Rho'	Lambda	Asbot	b	d	Rho'	Lambda
1	Right	----	----	----	0.000	2.000	----	----	----	0.000	2.000
2	Midspan	----	----	----	0.000	2.000	----	----	----	0.000	2.000
3	Midspan	----	----	----	0.000	2.000	----	----	----	0.000	2.000
4	Midspan	----	----	----	0.000	2.000	----	----	----	0.000	2.000
5	Left	----	----	----	0.000	2.000	----	----	----	0.000	2.000

NOTES: Deflection multiplier, Lambda, depends on moment sign at sustained load level and Rho' in given zone.
Rho' is assumed zero because Compression Reinforcement option is NOT selected in Solve Options.

Long-term Middle Strip Deflection Factors

Time dependant factor for sustained loads = 2.000
Units: Astop, Asbot (mm^2), b, d (mm), Rho' (%), Lambda (-)

		M+ve					M-ve				
Span	Zone	Astop	b	d	Rho'	Lambda	Asbot	b	d	Rho'	Lambda
1	Right	----	----	----	0.000	2.000	----	----	----	0.000	2.000
2	Midspan	----	----	----	0.000	2.000	----	----	----	0.000	2.000
3	Midspan	----	----	----	0.000	2.000	----	----	----	0.000	2.000
4	Midspan	----	----	----	0.000	2.000	----	----	----	0.000	2.000
5	Left	----	----	----	0.000	2.000	----	----	----	0.000	2.000

NOTES: Deflection multiplier, Lambda, depends on moment sign at sustained load level and Rho' in given zone.
Rho' is assumed zero because Compression Reinforcement option is NOT selected in Solve Options.

Extreme Long-term Column Strip Deflections and Corresponding Locations

Units: D (mm), x (m)

Span	Direction	Value	cs	cs+lu	cs+l	Total
1	Down	Def	---	---	---	---
		Loc	---	---	---	---
	Up	Def	-0.07	-0.13	-0.13	-0.17
		Loc	0.000	0.000	0.000	0.000
2	Down	Def	0.73	2.27	2.27	2.64
		Loc	2.527	2.601	2.601	2.601
	Up	Def	---	---	---	---
		Loc	---	---	---	---
3	Down	Def	0.34	1.05	1.05	1.22
		Loc	2.750	2.750	2.750	2.750
	Up	Def	-0.01	-0.01	-0.01	-0.02
		Loc	0.299	0.225	0.225	0.299
4	Down	Def	0.73	2.27	2.27	2.64
		Loc	2.973	2.899	2.899	2.899
	Up	Def	---	---	---	---
		Loc	---	---	---	---
5	Down	Def	---	---	---	---
		Loc	---	---	---	---
	Up	Def	-0.07	-0.13	-0.13	-0.17
		Loc	0.225	0.225	0.225	0.225

NOTES: Incremental deflections due to creep and shrinkage (cs) based on sustained load level values.
Incremental deflections after partitions are installed can be estimated by deflections due to:
- creep and shrinkage plus unsustained live load (cs+lu), if live load applied before partitions,
- creep and shrinkage plus live load (cs+l), if live load applied after partitions.
Total deflections consist of dead, live, and creep and shrinkage deflections.

Extreme Long-term Middle Strip Deflections and Corresponding Locations

Units: D (mm), x (m)

Span	Direction	Value	cs	cs+lu	cs+l	Total
1	Down	Def	---	---	---	---
		Loc	---	---	---	---
	Up	Def	-0.07	-0.13	-0.13	-0.17
		Loc	0.000	0.000	0.000	0.000
2	Down	Def	1.06	3.60	3.60	4.13
		Loc	2.601	2.676	2.676	2.676
	Up	Def	---	---	---	---

		Loc	---	---	---	---
3	Down	Def	1.01	2.85	2.85	3.36
		Loc	2.750	2.750	2.750	2.750
	Up	Def	-0.01	-0.01	-0.01	-0.01
		Loc	0.225	0.225	0.225	0.225
4	Down	Def	1.06	3.60	3.60	4.13
		Loc	2.899	2.824	2.824	2.824
	Up	Def	---	---	---	---
		Loc	---	---	---	---
5	Down	Def	---	---	---	---
		Loc	---	---	---	---
	Up	Def	-0.07	-0.13	-0.13	-0.17
		Loc	0.225	0.225	0.225	0.225

NOTES: Incremental deflections due to creep and shrinkage (cs) based on sustained load level values.
Incremental deflections after partitions are installed can be estimated by deflections due to:
- creep and shrinkage plus unsustained live load (cs+lu), if live load applied before partitions,
- creep and shrinkage plus live load (cs+l), if live load applied after partitions.
Total deflections consist of dead, live, and creep and shrinkage deflections.

7. Summary and Comparison of Design Results

Table 9 - Comparison of Moments obtained from Hand (EFM) and spSlab Solution (kN.m)			
		Hand (EFM)	**spSlab**
Exterior Span			
Beam Strip	Exterior Negative*	87.39	90.42
	Positive	68.03	64.33
	Interior Negative*	99.96	108.59
Column Strip	Exterior Negative*	0.00	0.00
	Positive	21.47	20.30
	Interior Negative*	31.55	34.27
Middle Strip	Exterior Negative*	0.00	0.00
	Positive	33.55	31.72
	Interior Negative*	49.29	53.55
Interior Span			
Beam Strip	Interior Negative*	91.56	98.62
	Positive	54.17	49.05
Column Strip	Interior Negative*	28.90	31.13
	Positive	17.10	15.48
Middle Strip	Interior Negative*	45.15	48.64
	Positive	26.71	24.19
* negative moments are taken at the faces of supports			

Table 10 - Comparison of Reinforcement Results							
Span Location		**Reinforcement Provided for Flexure**		**Additional Reinforcement Provided for Unbalanced Moment Transfer***		**Total Reinforcement Provided**	
		Hand	**spSlab**	**Hand**	**spSlab**	**Hand**	**spSlab**
Exterior Span							
Beam Strip	Exterior Negative	2 – 15M	2 – 25M	n/a	n/a	2 – 15M	2 – 25M
	Positive	2 – 15M	2 – 25M	n/a	n/a	2 – 15M	2 – 25M
	Interior Negative	2 – 15M	2 – 25M	---	---	2 – 15M	2 – 25M
Column Strip	Exterior Negative	6 – 15M	6 – 15M	12 – 15M	12 – 15M	18 – 15M	18 – 15M
	Positive	6 – 15M	6 – 15M	n/a	n/a	6 – 15M	6 – 15M
	Interior Negative	6 – 15M	6 – 15M	---	---	6 – 15M	6 – 15M
Middle Strip	Exterior Negative	9 – 15M	9 – 15M	n/a	n/a	9 – 15M	9 – 15M
	Positive	9 – 15M	9 – 15M	n/a	n/a	9 – 15M	9 – 15M
	Interior Negative	9 – 15M	9 – 15M	n/a	n/a	9 – 15M	9 – 15M
Interior Span							
Beam Strip	Positive	2 – 15M	2 – 25M	n/a	n/a	2 – 15M	2 – 25M
Column Strip	Positive	6 – 15M	6 – 15M	n/a	n/a	6 – 15M	6 – 15M
Middle Strip	Positive	9 – 15M	9 – 15M	n/a	n/a	9 – 15M	9 – 15M

Table 11 - Comparison of Beam Shear Reinforcement Results		
Span Location	**Reinforcement Provided**	
	Hand	**spSlab**
End Span		
Exterior	6 – 10M @ 280 mm	6 – 10M @ 281 mm
Interior	6 – 10M @ 280 mm	6 – 10M @ 281 mm
Interior Span		
Interior	8 – 10M @ 280 mm	8 – 10M @ 281 mm

Table 12 - Comparison of Two-Way (Punching) Shear Check Results (around Columns Faces)										
Support	b_1, mm		b_2, mm		b_o, mm		V_f, kN		c_{AB}, mm	
	Hand	spSlab	Hand	spSlab	Hand	spSlab	Hand	spSlab	Hand	spSlab
Exterior	514	514	577	577	1604	1604	199.5	215.3	230.4	230.4
Interior	577	577	577	577	2308	2308	458.0	460.4	288.5	288.5
Support	J_c, mm^4		γ_v		M_{unb}, kN.m		v_u, MPa		v_c, MPa	
	Hand	spSlab	Hand	spSlab	Hand	spSlab	Hand	spSlab	Hand	spSlab
Exterior	3.94×10^{10}	3.94×10^{10}	0.386	0.311	113.19	119.09	0.538	0.519	1.115	1.115
Interior	4.50×10^{10}	4.50×10^{10}	0.400	0.400	19.00	22.74	0.639	0.652	2.201	1.201

Table 13 - Comparison of Immediate Deflection Results (mm)								
Column Strip								
Span	D		D+LL$_{sus}$		D+LL$_{full}$		LL	
	Hand	spSlab	Hand	spSlab	Hand	spSlab	Hand	spSlab
Exterior	0.35	0.36	0.35	0.36	2.50	1.91	2.14	1.54
Interior	0.16	0.17	0.16	0.17	0.70	0.88	0.55	0.71
Middle Strip								
Span	D		D+LL$_{sus}$		D+LL$_{full}$		LL	
	Hand	spSlab	Hand	spSlab	Hand	spSlab	Hand	spSlab
Exterior	0.52	0.53	0.52	0.53	3.66	3.07	3.15	2.54
Interior	0.48	0.50	0.48	0.50	2.15	2.35	1.67	1.85

Table 14 - Comparison of Time-Dependent Deflection Results						
Column Strip						
Span	λ_Δ		Δ_{cs}, in.		Δ_{total}, in.	
	Hand	spSlab	Hand	spSlab	Hand	spSlab
Exterior	2.0	2.0	0.706	0.73	3.203	2.64
Interior	2.0	2.0	0.312	0.34	1.015	1.22
Middle Strip						
Span	λ_Δ		Δ_{cs}, in.		Δ_{total}, in.	
	Hand	spSlab	Hand	spSlab	Hand	spSlab
Exterior	2.0	2.0	1.03	1.06	4.70	4.13
Interior	2.0	2.0	0.96	1.01	3.11	3.36

In all of the hand calculations illustrated above, the results are in close or exact agreement with the automated analysis and design results obtained from the spSlab model. Excerpts of spSlab graphical and text output are given below for illustration.

8. Conclusions & Observations

A slab system can be analyzed and designed by any procedure satisfying equilibrium and geometric compatibility. Three established methods are widely used. The requirements for two of them are described in detail in CSA A.23.3-14 Clause 13.

Direct Design Method (DDM) is an approximate method and is applicable to two-way slab concrete floor systems that meet the stringent requirements of ***CSA A.23.3-14 (13.9.1)***. In many projects, however, these requirements limit the usability of the Direct Design Method significantly.

The Elastic Frame Method (EFM) does not have the limitations of Direct Design Method. It requires more accurate analysis methods that, depending on the size and geometry can prove to be long, tedious, and time-consuming.

StucturePoint's spSlab software program solution utilizes the Elastic Frame Method to automate the process providing considerable time-savings in the analysis and design of two-way slab systems as compared to hand solutions using DDM or EFM.

Finite Element Method (FEM) is another method for analyzing reinforced concrete slabs, particularly useful for irregular slab systems with variable thicknesses, openings, and other features not permissible in DDM or EFM. Many reputable commercial FEM analysis software packages are available on the market today such as spMats. Using FEM requires critical understanding of the relationship between the actual behavior of the structure and the numerical simulation since this method is an approximate numerical method. The method is based on several assumptions and the operator has a great deal of decisions to make while setting up the model and applying loads and boundary conditions. The results obtained from FEM models should be verified to confirm their suitability for design and detailing of concrete structures.

The following table shows a general comparison between the DDM, EFM and FEM. This table covers general limitations, drawbacks, advantages, and cost-time efficiency of each method where it helps the engineer in deciding which method to use based on the project complexity, schedule, and budget.

Applicable CSA A23.3-14 Provision	Limitations/Applicability	Concrete Slab Analysis Method		
		DDM (Hand)	EFM (Hand//spSlab)	FEM (spMats)
13.8.1.1 13.9.1.1	Panels shall be rectangular, with ratio of longer to shorter panel dimensions, measured center-to-center supports, not exceed 2.	☑	☑	
13.8.1.1 13.9.1.1	For a panel with beams between supports on all sides, slab-to-beam stiffness ratio shall be satisfied for beams in the two perpendicular directions.	☑	☑	
13.8.1.1 13.9.1.1	Column offset shall not exceed 20% of the span in direction of offset from either axis between centerlines of successive columns	☑	☑	
13.8.1.1 13.9.1.1	The reinforcement is placed in an orthogonal grid.	☑	☑	
13.9.1.2	Minimum of three continuous spans in each direction	☑		
13.9.1.3	Successive span lengths measured center-to-center of supports in each direction shall not differ by more than one-third the longer span	☑		
13.9.1.4	All loads shall be due to gravity only	☑		
13.9.1.4	All loads shall be uniformly distributed over an entire panel (q_f)	☑		
13.9.1.4	Factored live load shall not exceed two times the factored dead load	☑		
13.10.6	Structural integrity steel detailing	☑	☑	☑
13.10.10	Openings in slab systems	☑	☑	☑
8.2	Concentrated loads	Not permitted	☑	☑
13.8.4.1	Live load arrangement (Load Patterning)	Not required	Required	Engineering judgment required based on modeling technique
13.10.2*	Reinforcement for unbalanced slab moment transfer to column (M_{sc})	Moments @ support face	Moments @ support centerline	Engineering judgment required based on modeling technique
13.8.2	Irregularities (i.e. variable thickness, non-prismatic, partial bands, mixed systems, support arrangement, etc.)	Not permitted	Engineering judgment required	Engineering judgment required
Complexity		Low	Average	Complex to very complex
Design time/costs		Fast	Limited	Unpredictable/Costly
Design Economy		Conservative (see detailed comparison with spSlab output)	Somewhat conservative	Unknown - highly dependent on modeling assumptions: 1. Linear vs. non-linear 2. Isotropic vs non-isotropic 3. Plate element choice 4. Mesh size and aspect ratio 5. Design & detailing features
General (Drawbacks)		Very limited applications	Limited geometry	Limited guidance non-standard application (user dependent). Required significant engineering judgment
General (Advantages)		Very limited analysis is required	Detailed analysis is required or via software (e.g. spSlab)	Unlimited applicability to handle complex situations permissible by the features of the software used (e.g. spMats)

* The unbalanced slab moment transferred to the column M_{sc} (M_{unb}) is the difference in slab moment on either side of a column at a specific joint. In DDM only moments at the face of the support are calculated and are also used to obtain M_{sc} (M_{unb}). In EFM where a frame analysis is used, moments at the column center line are used to obtain M_{sc} (M_{unb}).